Analysis of the Relationship

between the Physical and the Mathematical Kinematic Parameters

in Robotic Manipulator Parameter Estimation Algorithms

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LIST OF SYMBOLS

d_i	joint variable, prismatic, joint i
d_{0i}	parameter, joint transducer zero-point offset, prismatic, joint i
$\mathbf{D}$	transformation matrix, used in gradient methods of optimization
$\mathbf{f}$	residual vector
$\mathbf{g}$	gradient vector
$\mathbf{H}$	Hessian matrix
$\mathbf{I}$	identity matrix
$\mathbf{J}$	Jacobian matrix
k_i	parameter, joint transducer output slope, joint i
n	total number of independent kinematic parameters
n_p	number of prismatic joints in a manipulator
n_r	number of revolute joints in a manipulator
n_s	number of calibration sites per posture
N	total number of independent kinematic and electrical parameters
$\mathbf{p}$	parameter vector
$\Delta\mathbf{p}$	parameter update vector
t	scalar, determines step length in gradient method of optimization
$\mathbf{T}$	transformation matrix, aggregate
$\mathbf{T}_{i,}{}^{J}$	transformation matrix, joint i
$\mathbf{T}_{i,}{}^{L}$	transformation matrix, link i
V_i	joint transducer output voltage, joint i
x_G	global x-axis
x_i	parameter, translation along local x-axis, link i; also, the axis itself

y_G	global y-axis
y_i	parameter, translation along local y-axis, link i; also, the axis itself
z_G	global z-axis
z_i	parameter, translation along local z-axis, link i; also, the axis itself
α_i	parameter, rotation about local z-axis, joint i
β_i	parameter, rotation about local y-axis, joint i
γ_i	parameter, rotation about local x-axis, joint i
θ_i	joint variable, revolute, joint i
θ_{0i}	parameter, joint transducer zero-point offset, revolute, joint i
ϕ	sum-of-squares of residuals (the objective function)

ABSTRACT

Analysis of the Relationship
between the Physical and the Mathematical Kinematic Parameters
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Although kinematic parameter estimation is well-established as a technique for improving end effector positioning accuracy of a robotic manipulator, little attention has been given to the relationship between the optimal mathematical parameters and the corresponding physical parameters of the manipulator. Such a relationship is very much desirable from the standpoint of preventive maintenance and isolation of sources of damage in the manipulator body. In this work, choice of kinematic model is shown to have a strong effect on this relationship, as well as on the calibration process in general. It is observed that the most desirable kinematic model for a given manipulator includes redundant parameters, which interact numerically among themselves during the solution process and complicate interpretation of results. A kinematic model with no redundant parameter, on the other hand, contains too few parameters to get a comprehensible relation between the physical dimensions of the manipulator and the mathematical parameters. Multiple point data collection in a single posture is shown to be of no significant help in preserving the mathematical/physical parameter relationship.

Chapter 1

INTRODUCTION

1.1 Problem Background

Kinematic positioning accuracy is one of the foremost concerns in the design of robotic manipulators, and consequently has received wide attention from the research community since the emergence of the robots with off-line programming capability. Accuracy, in this context, can be described as the maximum difference between the pose of the end of a manipulator's arm, as calculated using the controller's kinematic model, and the true pose of the end of the arm. One approach to improving accuracy is to specify tighter manufacturing tolerances on a manipulator's components. This approach, however, does not provide a long lasting solution and, moreover, leads to considerable increases in manufacturing costs. The end user, therefore, is often forced to compromise between cost and accuracy. Calibration, a second approach to improving accuracy, is in many ways preferable to the first, as described below.

The kinematic accuracy of a robotic manipulator depends on the accurate knowledge of the parameters which describe the manipulator's mechanical links and joint transducers. For various practical reasons, an accurate estimate of some of the parameters may not be available, and, in this case, numerical re-evaluation of these parameters in a calibration scheme typically produces greatly improved end effector positioning accuracy. The basic aim of robot calibration is to replace the initially estimated set of kinematic parameters with a new set of "optimal" parameters which reduces the aggregate end effector positioning error over the calibration space. To achieve this goal, the optimized parameters do not need to have any particular relationship to the actual physical parameters, and often, in fact, do not.

In the current work, the possibility of establishing a relationship between the set of optimal (mathematical) parameters and the actual (physical) dimensions of the manipulator's kinematic components is explored. Knowledge of this relationship will be important for establishing approaches to the analytical tracking of component wear or damage in predictive maintenance programs for robotic manipulators. In a predictive maintenance scheme, the optimal parameters could be included in a time-history database and the behavior of each of them observed over time for the purpose of identifying those parameters whose values are changing more than an expected or acceptable amount.

Choice of kinematic model is shown to have an important effect on both the robustness of the calibration algorithm and the mathematical/physical parameter relationship. Numerical simulations, utilizing both actual experimental data and synthesized data, have been run to compare several kinematic model/optimization algorithm combinations. These simulations have brought to light a number of properties of the kinematic models and the optimization algorithms which have important effects on the mathematical/physical parameter relationship. These properties will be useful in current efforts to refine this relationship for applications in predictive maintenance.

The following definitions will be used throughout this work:

- **position**: the translational relationship of one coordinate frame to another
- **attitude**: the rotational relationship of one coordinate frame to another
- **pose**: the position and attitude of one coordinate frame with respect to another
- **posture**: the unique sequential combination of a set of manipulator joint variables

- **parameter**: a quantity which describes some physical or mathematical aspect of a manipulator's kinematic model, assumed to be constant in magnitude over some useful period of time

- **global coordinate frame**: coordinate frame fixed to the ground with respect to which the pose of the end effector coordinate frame (EECF) is measured

- **redundancy**: the situation in which a kinematic model of a manipulator contains more parameters than are needed to describe the change in the pose of its EECF

- **site**: a point on a manipulator body where pose measurements are taken externally

1.2 Literature Survey

The technique of calibration has been adopted by many researchers as a primary approach to dealing with the problem of end effector coordinate frame (EECF) pose error. A calibration scheme may be formulated as a dimensional synthesis of manipulator parameters based on data taken from external measurements (Sommer and Miller, 1981). The functional inputs (manipulator joint transducer signals) must be recorded along with the desired functional outputs (actual poses of the EECF). An iterative numerical optimization routine may then be used to adjust manipulator parameters so as to minimize the aggregate error of the EECF pose over the entire workspace of the manipulator.

From a kinematic standpoint, parameter estimation may be expressed as a multiple-point synthesis of a spatial open-loop mechanism for rigid-body guidance. Similar syntheses of closed loop spatial linkages have been performed by Tull and Lewis (1968) and

Chen and Chan (1974). One of the restricted-step nonlinear least-squares optimization schemes popularly known as the Levenberg-Marquardt algorithm has been shown to work well for closed-loop mechanisms. Based on this knowledge, an efficient algorithm for simultaneous optimization (re-evaluation of the entire set of parameters) was developed by Sommer and Miller (1981) for the calibration of instrumented spatial linkages.

Several varying approaches to kinematic model selection and static calibration of robotic manipulators have been proposed. Whitney et al. (1984) were the first to publish an investigation of robot calibration which included actual measurement results. Their approach was unique in that it included the effects of non-geometric errors in the calibration process. More recently, Stone et al. (1986) presented a calibration method which utilized a different kinematic model from previous works. In these results, the authors found "dramatic" differences between optimized parameters and the corresponding physical manipulator parameters. Kirchner et al. (1987) proposed a method of calibration based on the Denavit-Hartenberg notation (Denavit and Hartenberg, 1955) which utilizes a parameter perturbation approach to avoid the problem of redundancy in the parameter set. These authors outlined an approach to deal with parameter redundancy through careful selection of the calibration postures.

According to Roth et al. (1987), manipulator calibration techniques may contain up to three different sublevels. At the first sublevel, the *joint sublevel*, calibration deals with the determination of the parameters describing joint transducers. At the second sublevel, the *kinematic model sublevel*, calibration deals with the determination of the geometric parameters describing the links and certain non-geometric parameters describing phenomena such as gear backlash and joint compliance. The third sublevel of calibration deals with the determination of the inertial properties of the various links and is known as the *dynamic sublevel*. The calibration techniques dealing with the first two sublevels are sometimes called *static calibration* or *kinematic parameter estimation*.

The scope of this work includes the first two sublevels, although in the present form it does not take into account the difficult-to-model non-geometric errors of the second sublevel. The non-geometric errors are caused by link compliance, gear train compliance, motor-bearing wobble, gear backlash etc. and in the present kinematic model they are assumed to be negligible. Judd and Knasinski (1987) observed that about 95% of the RMS value of the EECF positioning error is caused by errors in the geometric parameters. Therefore the geometric parameters of the links along with the electrical parameters of the joint transducers serve to model a manipulator almost completely. Manipulators are made up of a wide variety of driving mechanisms and it is extremely difficult to develop a model or a set of models which adequately describes the effects of the non-geometric parameters for an arbitrary manipulator. The cost and complexity involved in analyzing these effects on each individual manipulator after it is manufactured do not appear to be practical (Stone, 1987). Also, it has been assumed here that the resolution of the encoders providing joint position feedback is infinite.

In all of the references described above, the process of calibration was aimed primarily at reducing the end effector positioning error. The relationship between a physical parameter and its optimized value was first considered by Bosnik (1986). Aside from extending and applying the algorithm developed by Sommer and Miller (1981), this work described the potential application of the mathematical/physical parameter relationship in identifying and isolating points of damage or wear before such problems might become discernible by other means. If a static calibration is performed on a robotic manipulator at regular time intervals, then the values of the various parameters can be compared to their corresponding values during previous tests, for the purpose of identifying those parameters whose values are changing by more than an acceptable or expected amount. After additional experience with the procedure, it may be possible to make specific maintenance or repair recommendations for a manipulator on the basis of its calibration history. The mathematical/physical parameter relationship is obtained as an outcome of the calibration process itself, and no extra experimental setup or calculation is required. All that is needed is to modify the calibration algorithm in such a way that the results can be conveniently interpreted. It is an objective of the current work to develop a calibration technique which more closely relates the mathematical (theoretical) to the physical (actual) parameters.

1.3 Problem Statement

The primary objective of this study is to investigate the relationship between the physical features of the links and the joints of a manipulator and the parameters included in the kinematic model for that manipulator. A secondary objective is to be able to track the changes in these physical features from time to time by comparing the earlier values and the most recent optimal values which are obtained through parameter estimation algorithm in a robot calibration technique.

Specifically, meeting these objectives involves :

1. modification of an existing parameter estimation algorithm (Bosnik, 1986) to enable it to calculate the singular values and/or eigenvalues of the Jacobian and of the Hessian matrices of the manipulator system,

2. testing of the performance of the modified algorithm using synthesized data with known performance criteria,

3. interpretation of the results,

4. investigation of the characteristics of the objective function with respect to various kinematic and electrical parameters,

5. development of an algorithm accepting pose error data from multiple sites in a single posture, and

6. final interpretation of the results with reference to the extent of redundancy in a kinematic model.

1.4 Chapter Summary

This chapter describes the motivation for the present work and includes a literature survey which outlines the work of previous researchers in the area of the present work. The scope of this work is explained and the specific sub-problems to be tackled are listed.

Chapter 2
KINEMATIC MODELING

2.1 Introduction

A kinematic model can be defined as a systematic mathematical relationship between manipulator joint positions and the pose of the EECF. Coordinate frames are attached to different suitable places in the manipulator body and a relationship between two successive frames is obtained from manipulator geometry. The local relationships between the frames are concatenated, typically by matrix transformation methods, to get a relationship between the global coordinate frame and the EECF of the manipulator. Variables are included in the relationship so that any change in the EECF pose due to joint movements can be properly described. The inputs to a kinematic model are therefore the joint variables and the output is the EECF pose with respect to the global coordinate frame. The choice of a kinematic model is very important in calibration, often determining the success or failure of a particular calibration scheme.

2.2 Kinematic Models for Parameter Estimation

The choice of a kinematic model also impacts the desired mathematical/physical parameter relationship. Everett et al. (1987) attempted to categorize the desirable properties of a kinematic model to be used for calibration of manipulators having revolute and prismatic joints only. Three main properties were defined. First, the model should contain a sufficient number of parameters to express any possible variation in the kinematic structure of the robot; models possessing this property are *complete*. Second, the model should have a clear functional relationship to other acceptable models; models possessing this property are *equivalent*. Third, a small variation in the geometry of the robot should effect small only changes in the model parameters; models possessing this property are *proportional*. It was mentioned in the same paper that the properties of completeness and proportionality in a kinematic model are necessary for the model to be used in a general calibration scheme. These definitions of completeness, equivalence and proportionality are also useful in describing the ability of a kinematic model to preserve the physical/mathematical parameter relationship.

Although manipulator links are considered to be perfectly rigid in all current modeling methods, the process of parameter estimation may be visualized as allowing *any possible variation* to the *mathematically* flexible links and/or joint transducers so that the aggregate end effector positioning error is reduced. "Any possible variation" implies any change in the dimensions or characteristics of a manipulator component which render the features of that component different from its ideal or initially estimated features. The optimized parameter values obtained as a result of the calibration (optimization) process are again attributed to rigid manipulator components, and the pose of the EECF is subsequently calculated in terms of those constant parameters. The quality of completeness is related to the capability of a kinematic model to describe "any possible variation" in the physical kinematic structure.

In the present work, the objective is to relate the optimized value of the parameters to the physical features of the links and the joints of the manipulator. It is expected that a change in a dimension of a component will be reflected as a change in its corresponding mathematical parameter. Thus it has to be ensured that the kinematic model actually describes the physical dimensions and that each possible independent variation of the robot components are modeled separately. This is achieved by considering a link in three-dimensional space with a coordinate frame attached to each of its two ends. Keeping one frame fixed, the pose of the other frame can be changed in six independent ways — three corresponding to translation (along the fixed frame x, y, and z axes) and three corresponding to rotation (about the fixed frame x, y, and z axes). In other words, a combination of six primitive transformations is needed to compose the total transformation between the two frames (Stone, 1987). These are the kinematic features of the link under consideration.

Each of the six kinematic features described above is independent in the sense that each has an effect of its own and cannot be duplicated by any combination of the other five. Mathematically speaking, the complete relationship between two unconstrained coordinate frames can be represented by a vector in a six-dimensional space. The translational and rotational directions are represented by the six basis vectors of that six-dimensional space. For any model to be able to reflect the change in any of the six kinematic features of a link, it has to have at least six parameters (a necessary condition) spanning the six-dimensional space (a sufficient condition) described above. Zeigert and Datseris (1988) arrived at a similar conclusion .

It is also to be understood that the modeling must be completely unconstrained. Constrained models may describe the pose of the end effector with a smaller number of parameters, but they fail to describe each of the kinematic features of the links separately. This prevents the gaining of a clear picture of the mathematical/physical parameter relationship.

The quality of proportionality in a kinematic model affects both the stability of the numerical optimization algorithm and the interpretation of the optimized parameters. It has been noted (Everett, 1987; Hayati, 1983) that the Denavit-Hartenberg notation lacks this property in case where the model includes two parallel or nearly parallel joints. Proportionality of a kinematic model is an outgrowth of the physical nature of a manipulator, and if the parameters of a model depict, at least to some extent, the physical dimensions of the links, then proportionality of the kinematic model will be inherently achieved.

It is interesting to note that most of today's popular kinematic models do not satisfy the fundamental requirements above. The ubiquitous Denavit-Hartenberg model uses only three parameters per link (namely, link length, link twist, and joint offset), and therefore is not generally a useful model for calibration purposes. The more closely the mathematical parameters relate to the physical shape of a link, the more closely will the optimized parameters tend to relate to the true shape of the link. A better description of the mathematical/physical parameter relationship will be obtained if the line joining the two coordinate frames travels through the body of the link instead of traveling through space. Although achieving this goal may be difficult for any arbitrarily shaped link, for most common industrial robots this condition is ensured by merely establishing the coordinate frame origins inside the body of the robot. Figure 2.1 illustrates two typical coordinate frame choices, one of which reflects the kinematic features of the link (the unprimed frames) and the other which does not (the primed frames). When such information has been lost due to choice of coordinate frame locations, it cannot be regained through calibration.

Figure 2.1: Two Typical Coordinate Frame Attachments

2.3 Description of a Suitable Model

The kinematic notations proposed by Sheth and Uicker (1971) are well-suited for calibration. In the current work a similar model used by Bosnik (1986) is adopted. It is closely related to that used by Sheth and Uicker, is easy to understand in terms of mathematical/physical relationships, and is simple to assign to the kinematic chains. Figure 2.2 summarizes the notation used in this work. Under this system, the link transformation matrix is

T^L = \B\BC\[(\A\AC\VS9\HS16\CO4(cαcβ,cαsβsγ-sαcγ,cαsβcγ+sαsγ,x,sαcβ,sαsβsγ+cαcγ,sαsβcγ-cαsγ,y,-sβ,cβsγ,cβcγ,z,0,0,0,1)) , (2.1)

where x, y, and z represent the position of the distal coordinate frame origin with respect to the proximal coordinate frame; α, β, and γ represent a set of ZYX-ordered Euler angles relating the distal coordinate frame to the proximal frame; and cα = cosine(α), sα = sine(α), etc.

The manipulator joint variables must be measured accurately during the course of an experimental session. For the revolute and prismatic joints commonly found in robotic manipulators, joint transducers in common use include potentiometers, synchros, resolvers, contact encoders, and non-contact magnetic and optical encoders. The number of parameters chosen for each joint depends upon the nature of the characteristic curve of the transducer at that particular joint. Assuming all transducers to be linear in nature, we must add two parameters per joint, expressing the joint variable for joint "i" in terms of its parameters as

Figure 2.2: Summary of Kinematic Notation

$$d_i = d_{0i} + k_i V_i \tag{2.2}$$

for prismatic joints, or

$$\theta_i = \theta_{0i} + k_i V_i \tag{2.3}$$

for revolute joints, where d_0 and θ_0 are the joint transducer zero point offsets, k is the joint transducer output slope, and V is the joint transducer output voltage. The joint transducer model need not be linear, of course, and additional terms may be added as necessary to accommodate nonlinear transducer characteristics. The joint transformation matrices, then, are

$$\mathbf{T}_{p},^{J} = [\ 1,0,0,0,0,1,0,0,0,0,1,d,0,0,0,1\ ] \tag{2.4}$$

for prismatic joints, and

$$\mathbf{T}_{r},^{J} = [\ c\theta,-s\theta,0,0,s\theta,c\theta,0,0,0,0,1,0,0,0,0,1\ ] \tag{2.5}$$

for revolute joints, where again $c\theta$ = cosine(θ) and $s\theta$ = sine(θ).

The most general robotic manipulator is an open-loop six degree-of-freedom (DOF) kinematic chain composed of a base and an end effector connected by five intermediate links and six intermediate joints, as shown in Figure 2.3. Allowing six parameters per link and two parameters per joint, a total of 54 parameters is required to completely describe the manipulator *for calibration purposes* via this model. This, therefore, is the minimum set of parameters required to maintain the physical/mathematical parameter relationship. The aggregate transformation matrix relating the EECF to the global coordinate frame may then be expressed as a sequential multiplication of the 13 local homogeneous transformation matrices, as follows:

$$\mathbf{T} = \mathbf{T}_{1,}{}^{L}\,\mathbf{T}_{1,}{}^{J}\,\mathbf{T}_{2,}{}^{L}\,\mathbf{T}_{2,}{}^{J}\,\mathbf{T}_{3,}{}^{L}\,\ldots\,\mathbf{T}_{6,}{}^{L}\,\mathbf{T}_{6,}{}^{J}\,\mathbf{T}_{7,}{}^{L} \quad . \tag{2.6}$$

The global position and attitude of the EECF can be extracted from the matrix T above by any standard method found in textbooks (Fu et al., 1987).

2.4 Chapter Summary

In this chapter it was emphasized that the choice of a particular kinematic model is important in calibration. Often, the choice becomes crucial, determining the success and failure of the calibration scheme. To model all the kinematic features of a manipulator, six parameters are needed per link and two parameters per joint (considering linear joint transducer characteristics). A suitable model was described which possesses all the necessary qualities of a model to be used for a parameter estimation procedure.

Figure 2.3: General Six Degree-of-Freedom Robotic Manipulator

Chapter 3

LEAST SQUARES PARAMETER ESTIMATION TECHNIQUES

3.1 Introduction

The various methods of calibration which have been proposed differ mainly in the choice of their respective kinematic models. Regardless of the kinematic model chosen, some differences between the true pose of the EECF (as measured externally) and the pose predicted by the internal kinematic model will be observed for any real robot. These differences, known as residuals, will be observed at each calibration posture. The aggregate sum-of-squares of the residuals ϕ may be calculated as

$$\phi = \sum_{i=1}^{n} \left[(x_{Gi}^{e}-x_{Gi})^2+(y_{Gi}^{e}-y_{Gi})^2+(z_{Gi}^{e}-z_{Gi})^2+(\alpha_{Gi}^{e}-\alpha_{Gi})^2+(\beta_{Gi}^{e}-\beta_{Gi})^2+(\gamma_{Gi}^{e}-\gamma_{Gi})^2\right] , \quad (3.1)$$

where n is the number of calibration postures, the superscripted terms are the externally measured global position and attitude values, and the unsuperscripted terms are the global position and attitude values of the EECF as predicted by the internal kinematic model. The parameter values used by the kinematic controller of the manipulator may not be the parameters' actual physical values. The sum-of-squares of the residuals ϕ is the objective function, and it is the goal of the optimization to minimize this to ensure best possible performance of the manipulator. The set of parameters for which ϕ is minimum is the desired or optimal set.

In the procedure described above, data is collected at only one site on the manipulator body in a single posture, namely at the EECF. It is also possible to collect data at several different points on the manipulator in a single posture. In this latter case, the objective function will be the aggregate sum of the squares of the positional and orientational errors at all the sites and summed over all the postures.

In the above discussions it has implicitly been assumed that the external measurements are perfect. This assumption might lead to erroneous calibration results if the instruments used for external pose measurements contain errors. Considering the fact that the magnitude of accuracy in the measuring devices (sonic digitizer, as an example) are much more than those of the typical manipulators, it can be safely said that the minimization of the objective function (Equation 3.1) would contribute significantly to the kinematic accuracy of the manipulator.

Regardless of what external measuring method is chosen, three basic considerations must be satisfied as listed by Bosnik (1986): (1) positioning precision and knowledge of the relative position and attitude at each calibration posture must be significantly better than the desired manipulator accuracy, (2) measurement of the transducer output signals must be at least as accurate as desired manipulator accuracy, (3) a sufficient number of calibration postures should be measured so as to representatively populate and span the desired manipulator workspace. Also, the number of calibration postures should be greater than one-sixth the number of estimated parameters (if using all six pose quantities at each calibration posture, that is three radial vector components and three Euler angles).

3.2 Quadratic Model of The Objective Function

The multivariable function ϕ depends on all the 54 parameters describing the manipulator and it can be expressed as the second-order expansion of a Taylor series as

$$\phi(\mathbf{p}+\Delta\mathbf{p}) \cong \phi(\mathbf{p}) + \mathbf{g}^T\Delta\mathbf{p} + \frac{1}{2!}\,\Delta\mathbf{p}^T\mathbf{H}\Delta\mathbf{p} \quad , \tag{3.2}$$

where

$$\mathbf{p}^T = \{p_1 \quad p_2 \quad p_3 \quad \ldots \quad p_{54}\} \tag{3.3}$$

is the transpose of the vector of parameter initial guesses,

$$\Delta\mathbf{p}^T = \{\Delta p_1 \quad \Delta p_2 \quad \Delta p_3 \quad \ldots \quad \Delta p_{54}\} \tag{3.4}$$

is the transpose of the vector of parameter updates at each iteration,

$$\mathbf{g}^T = \nabla\phi = \left\{ \frac{\partial\phi}{\partial p_1} \quad \frac{\partial\phi}{\partial p_2} \quad \frac{\partial\phi}{\partial p_3} \quad \ldots \quad \frac{\partial\phi}{\partial p_{54}} \right\} \tag{3.5}$$

is the transpose of the gradient vector, and

$$\mathbf{H} = \nabla^2\phi = \begin{bmatrix} \frac{\partial^2\phi}{\partial p_1^2} & \frac{\partial^2\phi}{\partial p_1\partial p_2} & \frac{\partial^2\phi}{\partial p_1\partial p_3} & \ldots & \frac{\partial^2\phi}{\partial p_1\partial p_{54}} \\ \frac{\partial^2\phi}{\partial p_2\partial p_1} & \frac{\partial^2\phi}{\partial p_2^2} & \frac{\partial^2\phi}{\partial p_2\partial p_3} & \ldots & \frac{\partial^2\phi}{\partial p_2\partial p_{54}} \\ \frac{\partial^2\phi}{\partial p_3\partial p_1} & \frac{\partial^2\phi}{\partial p_3\partial p_2} & \frac{\partial^2\phi}{\partial p_3^2} & \ldots & \frac{\partial^2\phi}{\partial p_3\partial p_{54}} \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial^2\phi}{\partial p_{54}\partial p_1} & \frac{\partial^2\phi}{\partial p_{54}\partial p_2} & \frac{\partial^2\phi}{\partial p_{54}\partial p_3} & \ldots & \frac{\partial^2\phi}{\partial p_{54}^2} \end{bmatrix} \tag{3.6}$$

is the Hessian matrix, which is always square, symmetric, and quadratic. The gradient vector ideally has zero magnitude at the minimum of the objective function. The right-hand side of Equation 3.2 is the quadratic model of the actual function on the left-hand side. This model will be reasonably accurate if its application is restrained to a sufficiently small neighborhood of the current objective function value.

The Hessian matrix must be either positive definite or positive semi-definite. Geometrically, an n^{th}-order positive definite Hessian matrix represents a paraboloid in (n+1)-dimensional space and its contour represents an n-dimensional ellipsoid. In the special case of three-dimensional space, the surface will be a paraboloid opening toward the positive z-axis. For equal eigenvalues, the horizontal cross section (contour) of this paraboloid will be a circle, and for unequal eigenvalues, the cross section will be an ellipse. The shape of the ellipse will depend on the relative magnitudes of the eigenvalues, the major axis being in the direction of the eigenvector associated with the smaller eigenvalue. In any case, if the Hessian is positive definite, the associated surface as well as the associated function will have a definite minimum corresponding to a unique set of variables. The position of the minimum is at the vertex of the paraboloid.

If one of the system eigenvalues becomes zero, the major axis of the n-dimensional surface becomes infinitely long, and the Hessian is positive semi-definite. In three dimensions the surface will be an infinitely long parabolic trough. Thus, one of the vertical sections will be a straight line and another will be a parabola opening indefinitely towards the positive z-axis. The surface associated with a positive semi-definite matrix does have a definite minimum, but different combinations of dependent variables may yield the same minimum value. Once the bottom (minimum) of the trough is reached, no change in the magnitude of the function will be observed along any of the directions associated with zero eigenvalues. A positive semi-definite

Hessian is, of course, singular, and represents a system with an infinite number of valid solutions.

Another important matrix associated with a robotic system is the Jacobian matrix **J**. For a robotic system, a typical element of **J** expresses the change in the objective function at a particular posture with respect to a change in one of the parameters. It is also possible to express the elements of this matrix as a change in one of the global pose components (x_G, y_G, z_G, α_G, β_G, γ_G) of the EECF (as extracted from the aggregate transformation matrix) at a particular posture with respect to a change in one of the kinematic parameters. Recalling that there are 54 parameters in the proposed kinematic model for a 6-DOF manipulator, the Jacobian matrix *for a single posture* takes on the expanded form

$\mathbf{J}_i$ =

\B\BC\[(\A\AC\VS9\HS16\CO5(\F(∂x_{Gi},∂p_1),\F(∂x_{Gi},∂p_2),\F(∂x_{Gi},∂p_3),...,\F(∂x_{Gi},∂p_{54}),\F(∂y_{Gi},∂p_1),\F(∂y_{Gi},∂p_2),\F(∂y_{Gi},∂p_3),...,\F(∂y_{Gi},∂p_{54}),\F(∂z_{Gi},∂p_1),\F(∂z_{Gi},∂p_2),\F(∂z_{Gi},∂p_3),...,\F(∂z_{Gi},∂p_{54}),\F(∂α_{Gi},∂p_1),\F(∂α_{Gi},∂p_2),\F(∂α_{Gi},∂p_3),...,\F(∂α_{Gi},∂p_{54}),\F(∂β_{Gi},∂p_1),\F(∂β_{Gi},∂p_2),\F(∂β_{Gi},∂p_3),...,\F(∂β_{Gi},∂p_{54}),\F(∂γ_{Gi},∂p_1),\F(∂γ_{Gi},∂p_2),\F(∂γ_{Gi},∂p_3),...,\F(∂γ_{Gi},∂p_{54}))) , (3.7)

where "i" is the posture number. Each calibration posture adds 6 similar rows to the total system Jacobian matrix **J**. In almost all the cases, the Jacobian matrix is a rectangular matrix with number of rows exceeding the number of columns.

The definitions expressed in Equations 3.5 and 3.6 can now be rewritten in terms of the Jacobian as

$$\mathbf{g} = -2\mathbf{J}^T\mathbf{f} \tag{3.8}$$

where $\mathbf{f}$ is the vector of residuals, and

$$\mathbf{H} = 2\mathbf{J}^T\mathbf{J} \quad . \tag{3.9}$$

The above results can be found in most of the textbooks dealing with the minimization of least squares error (Fletcher, 1980; Nash, 1979). Subsequently in this work, the name Hessian is loosely used to represent the $\mathbf{J}^T\mathbf{J}$ matrix, although they are not exactly the same in the stricter sense.

3.3 Optimization Techniques

3.3.1 Steepest Descent Method

The steepest descent method is one of the simplest forms of gradient search methods. The gradient search methods, in general, are based on the fact that the gradient of a function points in the direction of its maximum rate of increase. Therefore, if the search direction is chosen to be opposite to that of the gradient, it will be pointed in the direction of the maximum rate of decrease of the function. This search technique continues iteratively, until the minimum of the function is reached.

As described by Nash (1979), the basic iteration step taken by most of the descent methods is

$$\Delta\mathbf{p} = -t\mathbf{D}\mathbf{g} \quad , \tag{3.10}$$

where $\mathbf{D}$ is a matrix defining a transformation of the gradient and t is a scalar describing the length of the step taken in the direction opposite to that of the gradient. For the steepest descent method, $\mathbf{D}$ is chosen to be an identity matrix and t is chosen in such a way that the objective function is reduced in that particular step. In particular, if t=1, the steepest descent step is determined as

$$\Delta \mathbf{p} = -\mathbf{g} \quad . \tag{3.11}$$

The elements of the update vector are added to the corresponding elements of the parameter vector to give the optimal set of parameters for that particular iteration. This process is continued until one or more pre-defined convergence criteria (corresponding to a minimum objective function value) are met. For a steepest descent method, it is not necessary to use the quadratic model of the objective function.

3.3.2 Gauss-Newton Algorithm

The Gauss-Newton algorithm is one of the most basic second-order optimization techniques for non-singular systems. It utilizes the fact that the magnitude of the gradient of a function at a minimum is zero. In this technique, the transformation matrix **D** in Equation 3.10 is replaced by $\mathbf{H}^{-1}$, the inverse of the Hessian matrix. Commonly, the value of t remains unity. The optimization scheme proceeds in an iterative way. In each iteration an update vector $\Delta\mathbf{p}$ is obtained via the equation

$$\Delta \mathbf{p} = -\mathbf{H}^{-1}\mathbf{g} \quad , \tag{3.12}$$

which minimizes the objective function in a small neighborhood of its current value. Using the relations in Equations 3.8 and 3.9, the above equation can be rewritten as

$$\Delta \mathbf{p} = (\mathbf{J}^T\mathbf{J})^{-1}(\mathbf{J}^T\mathbf{f}) \quad . \tag{3.13}$$

This is the *normal equation*, a well-known part of least squares estimation. Based on the kinematic model chosen for this work, Equation 3.13 represents the solution of 54 parameter updates in a single iteration. Since the Jacobian matrix is not square, Equation 3.13 cannot be written in any simpler form.

To ensure that the optimal parameter set will increase end effector positioning accuracy throughout the desired workspace, calibration data should be collected at a number of calibration postures sufficient to populate and span the workspace. This typically requires many more calibration postures than the minimum that would be needed for the evaluation of Equation 3.13 (Sommer and Miller, 1981).

3.3.3 Disadvantages of the Simpler Methods

Although the steepest descent method is guaranteed to reach a local minimum or a saddle point, the greatest disadvantage with this method is that it is often very inefficient. There are common cases in which the successive directions chosen by this method are nearly opposites, and which are both nearly perpendicular to the direction in which the minimum is to be found . This progression, sometimes called the hemstitching pattern, is due to the search directions which are not linearly independent (Nash, 1979). In computational terms, the steepest descent method requires an excessively large number of iterations for convergence and, therefore, is not appealing from a practical standpoint.

The Gauss-Newton method fails in case where the Hessian matrix is not positive definite. It was observed by Fletcher (1980) that, even in case of positive definite Hessians, convergence may not occur. For an n^{th}-order system, the Gauss-Newton method requires evaluation of n^2 second derivatives, n first derivatives, and also the inversion of a matrix. For a complicated function, these may be difficult to perform.

For a function with a positive semi-definite Hessian, Equation 3.13 will represent an infinite number of valid solutions.

3.3.4 Levenberg-Marquardt Algorithm

The difficulties encountered in the steepest descent method and in the Gauss-Newton methods are rectified in an algorithm developed by Levenberg (1944) and Marquardt (1963) which is popularly known as the *Levenberg-Marquardt algorithm*. This method has been applied successfully in various optimization schemes. According to this algorithm, Equation 3.13 should be modified as

$$\Delta \mathbf{p} = (\mathbf{J}^T\mathbf{J}+\lambda\mathbf{I})^{-1}(\mathbf{J}^T\mathbf{f}) \quad , \tag{3.14}$$

where λ is an adjustable scalar and **I** is an identity matrix of the same order as that of the Hessian. This approach essentially adds a positive quantity to each of the diagonal elements of the Hessian. A sufficiently large λ will always be available which will force the Hessian to be non-singular.

This method is a mixture of first order gradient search techniques and second order Gauss-Newton techniques. For parameter estimates far from optimum, the quantity λ typically increases and the method resembles a steepest descent technique. As the search comes closer to the minimum, λ typically becomes very small, making the search direction almost coincident with the Gauss-Newton search direction. This technique successfully minimizes the objective function.

On some occasions matrix **I** in Equation 3.14 is replaced by the diagonal matrix $\mathbf{B}^2$, where the elements of **B** are defined as

$$B_{ii} = \sqrt{H_{ii}} \qquad \text{for} \quad H_{ii} \neq 0$$
$$B_{ii} = 1 \qquad \text{for} \quad H_{ii} = 0 \quad . \tag{3.15}$$

This replacement provides a better scaling of the parameters in that it helps to improve the numerical conditioning of the system.

Geometrically, the Levenberg-Marquardt algorithm transforms the original surface in such a way that the unique minimum of the transformed surface is equal to the minimum (minima) of the original surface. Thus, good updates are obtained and relatively few steps are required to reach the minimum of the objective function.

3.4 Singular Value Decomposition

Singular value decomposition is one of the most robust mathematical approaches to solve a least squares problem. Although it can be directly applied to evaluate the solutions as represented in Equation 3.13, it is possible to avoid the calculations in Equations 3.8 and 3.9 and to solve a simpler system, as follows. Equation 3.13 can be rewritten in terms of the Jacobian matrix as

$$\mathbf{J}\Delta\mathbf{p} = \mathbf{f} \quad . \tag{3.16}$$

The above system of equations has a solution if **f** lies in the column space of **J**. For a typical manipulator, the vector representing the residuals does not lie in the column space of **J**. This means that, in general, there is no parameter update vector for which each of the residuals is identically equal to zero. Thus, a solution for Equation 3.16 is not obtained. This is due to the fact that all the kinematic features of the manipulator are not modeled and a re-evaluation of the modeled parameters might not always compensate for the unaccounted and unknown errors of the unmodeled ones. In such a case, the algorithm actually attempts to identify a parameter set such that the quantity $\|\mathbf{J\Delta p\text{-}f}\|$ is minimum (Strang, 1976). In terms of the three dimensional case, illustrated in Figure 3.1, the condition to be satisfied is that the vector joining the point P (where OP represents the optimal parameter set) and F (where OF represents the residual vector) must be perpendicular to the (planar) column space.

The uniqueness of a solution in a least squares scheme is ensured if the columns of the Jacobian are linearly independent. If they are not, the system has an infinite number of valid solutions. This is because the null space of the Jacobian matrix is non-zero. In this case the point P is still uniquely determined by being closest to F, but it can now be expressed as a non-unique combination of the columns of the Jacobian, and again an infinite number of solutions results.

One of the ways to deal with such a problem is to make a choice according to the rule that, among all solutions, the optimal solution (a vector) is the one that has the minimum magnitude. This can be achieved by the singular value decomposition technique. In this method, the $m \times n$ Jacobian matrix (where $m = 6$ times the number of postures, n = number of parameters, and $m > n$) can be factored into

$$\mathbf{J} = \mathbf{Q}_1 \mathbf{E}\, \mathbf{Q}_2{,}^T , \tag{3.17}$$

Figure 3.1: Least Squares Estimation Model (3-D Case)

where $\mathbf{Q}_1$ is an m×m orthogonal matrix, $\mathbf{Q}_2$ is an n×n orthogonal matrix and **E** is expressed as

$$\mathbf{E} = [\ \Sigma,\mathbf{0},\mathbf{0},\mathbf{0}\ \ ]\ , \tag{3.18}$$

where Σ = diagonal($\sigma_1,\ \sigma_2,\ \sigma_3,\ \ldots\ \sigma_r$), σ_i being the i^{th} singular value of the Jacobian matrix and r being the rank of the Jacobian. The inverse of the Jacobian is obtained by

$$\mathbf{J}^+ = \mathbf{Q}_2\mathbf{E}^+\mathbf{Q}_1,^T\ , \tag{3.19}$$

where $\mathbf{J}^+$ is called the pseudoinverse or the generalized inverse of the Jacobian and $\mathbf{E}^+$ is the pseudoinverse of **E.** $\mathbf{E}^+$ is obtained by merely replacing the singular values of **E** with their respective reciprocals. Equation 3.16 becomes

$$\Delta\mathbf{p} = \mathbf{J}^+\mathbf{f}\ \ . \tag{3.20}$$

In a similar way the pseudoinverse of $\mathbf{J}^T\mathbf{J}$ can also be calculated to evaluate Equation 3.13. For a symmetric matrix such as the Hessian, the columns of $\mathbf{Q}_2$ will be a set of orthonormal eigenvectors. It is general practice to set very small singular values to zero rather than to invert them; otherwise, the very large reciprocals of the small singular values overwhelm the pseudoinverse.

3.5 Chapter Summary

The least squares parameter estimation scheme was detailed in this chapter. The objective function was expanded as a second-order Taylor series and optimization techniques were employed to minimize this function. Two simple techniques, namely, the steepest descent method and the Gauss-Newton algorithm were described. These simpler methods, though easy to implement, suffer from some inherent difficulties. There are more efficient algorithms which are based on the simpler algorithms but which rectify most of their computational difficulties. Two such algorithms are the Levenberg-Marquardt algorithm and the singular value decomposition algorithm.

Chapter 4

SIMULATION, RESULTS, AND DISCUSSION

4.1 Introduction

Several kinematic model/calibration algorithm combinations have been investigated, using both actual external position data collected from the manipulator and synthesized position data for the same manipulator. The optimization algorithms were implemented in VAX FORTRAN (version 4.8) and were run on a VAX 11/750 minicomputer. Some subroutines were taken from the IMSL mathematical and statistical subroutine library.

4.2 Kinematic Description of the Manipulator

Most of the investigations were based on a General Electric Model A4 industrial manipulator. As shown in the Figure 4.1, the A4 is a 4 DOF manipulator of the RRPR type, having five links and four joints. Coordinate frames are attached to both ends of each of the links. The base coordinate frame can be suitably fixed at a point during calibration which is convenient for data collection. According to the kinematic model used in this work (discussed in Section 2.3) 38 parameters (30 geometric and 8 electrical) are needed to describe the A4 completely. The nominal values of these parameters are listed in Table 4.1, where the lengths are measured in inches and the angles are measured in degrees. The parameters are numbered in a sequence for easy reference (parameter numbers appear in parentheses following each parameter). Table 4.1 should be read as follows: the x-coordinate length of the first link is 15.080 inches and it is parameter p_1. In the ideal

Figure 4.1: Coordinate Frame Attachments for GE Model A4 Robot Manipulator

Table 4.1: Optimal Values of Parameters for Synthesized Data (Notations in Parentheses are Parameter Identifiers):

	Link 1	Link 2	Link 3	Link 4	Link 5
x (in) (p_{25})	15.080 (p_1)	0.000 (p_7)	0.000 (p_{13})	0.000 (p_{19})	0.875
y (in) (p_{26})	-8.500 (p_2)	15.750 (p_8)	11.810 (p_{14})	0.000 (p_{20})	-0.125
z (in) (p_{27})	10.540 (p_3)	0.000 (p_9)	0.000 (p_{15})	1.500 (p_{21})	3.000
α (deg) (p_{28})	0.000 (p_4)	0.000 (p_{10})	0.000 (p_{16})	0.000 (p_{22})	171.000
β (deg) (p_{29})	180.000 (p_5)	0.000 (p_{11})	0.000 (p_{17})	0.000 (p_{23})	0.000
γ (deg) (p_{30})	0.000 (p_6)	0.000 (p_{12})	0.000 (p_{18})	0.000 (p_{24})	90.000

	Joint 1	Joint 2	Joint 3	Joint 4
θ_{oi}	90.000 (p_{31}) (degrees)	-117.000 (p_{32}) (degrees)	0.250 (p_{33}) (inches)	200.000 (p_{34}) (degrees)
k_i	7.200 (p_{35}) (degrees/volt)	7.200 (p_{36}) (degrees/volt)	1.400 (p_{37}) (inches/volt)	9.000 (p_{38}) (degrees/volt)
TYPE	revolute	revolute	prismatic	revolute

model the four joint axes are parallel, regardless of posture, and this makes the A4 a rather interesting case for study, as will be described subsequently.

4.3 Synthesis of Data

The parameter estimation algorithms were tested on synthesized data. This approach provided a theoretical case for which the minimum value of the objective function ϕ was known to be zero and the corresponding optimal parameter set was known in advance. The optimal parameters were arbitrarily fixed to be the design parameters, since that set of parameters was readily available.

The forward kinematic program generated the position and orientation of the EECF for the optimal set of parameters and for an arbitrary set of joint voltages. This was repeated for about 30 different sets of voltages for which the end effector populated the workspace of the manipulator reasonably well. The next step was to perturb the values of some of the parameters to simulate the condition in which those parameters had undergone some distortion in course of the operation of the manipulator. The calibration algorithm was then run and it was observed whether the perturbed values returned to their original values when the objective function converged to zero. This allowed for a reasonable comparison of the performance of various kinematic models and optimization schemes.

In case of multiple sites per posture calibration scheme, positional and orientational data were collected not only at the EECF but also at some other convenient sites on the manipulator body. As the position and orientation of a site with respect to its nearest proximal coordinate frame are not known exactly, for each of the sites, six parameters are added to the original list of parameters. These site parameters are not very important in a calibration scheme because the knowledge of the exact position of a site with respect to the previous coordinate frame does not directly help in reducing the EECF positioning error. Nevertheless it is required to include those parameters in the optimization scheme to gain some indirect advantages which will be described later in this chapter.

4.4 Results and Discussion

4.4.1 Effect of Parameters on EECF Pose

If the repetitive multiplication of the matrices of the right hand side of Equation 2.6 is performed, the translational and rotational quantities can be extracted from the final transformation matrix on the left hand side. Each of the six pose components will be functions of the kinematic parameters. Although the functions are complex in nature, it is possible to get a good idea about the effect of the kinematic parameters on the pose of EECF. The functions describing these relations should in general change from one site to another in the manipulator workspace. A similar analysis was reported by Kirchner, et al. (1987).

Another way to view this situation is to draw the characteristic curves of each of the EECF pose components as they change with respect to the kinematic parameters. Although, at first sight, it appears to take a long time to draw all the 228 possible curves (for 38 parameters and 6 pose components), a pattern of behavior of the parameters begins to emerge after only a few curves are drawn. Therefore, to demonstrate the effect of variation of the kinematic parameters on the EECF pose component, only a small subset of all of the possible curves are required. These curves give instant visual representation of the nature of the characteristics of the parameters and of their effects on EECF pose.

A representative group of these curves are shown in the Figures 4.2, 4.3, and 4.4. The EECF pose components are plotted along the vertical axes and the parameters along the horizontal axes. When a certain EECF pose component does not depend on a certain parameter, its characteristic graph with respect to that parameter is a horizontal straight line. For each of the curves, the parameters are varied for 100 units, 50 units on each side of their nominal values. In the particular posture considered for drawing the above-mentioned figures, the joint voltages are -6.883V, 13.301V, 4.122V, and -28.645V for joints 1, 2, 3, and 4, respectively, and the corresponding joint variables are 40.443°, -21.233°, 6.021 inches, and -57.809°. With the help of forward kinematics, the coordinates of the origin of the EECF were found to be at 28.576, 13.995, and 0.020 (all in inches) with respect to the global origin. The attitude of the EECF, represented by ZYX-ordered Euler angles, was found to be 47.600°, 0°, and -90° respectively. Table 4.2 lists the joint variables and EECF pose components for 5 typical postures. Data from posture #4 of this table were selected for drawing the curves.

In each of Figures 4.2, 4.3, and 4.4, a vertical dotted line is drawn on the parameter axis at the nominal parameter value. Where it meets the curve, another line is drawn horizontally to meet the vertical axis at the corresponding EECF pose component value. The nominal value of a parameter and the corresponding pose component can also be obtained from forward kinematics. Thus we can compare the known analytical solution and the solution exhibited by a curve as a means of confirming the validity of the curve.

Figure 4.2 shows the characteristic curves for linear kinematic parameters. All the curves are seen to be linear. Also, the EECF attitudes are found to be independent of the linear parameters, as shown in Figures 4.2c and d.

Figure 4.2: EECF Pose Characteristic Curves for Linear Parameters

Figure 4.3: EECF Pose Characteristic Curves for Angular Parameters

Figure 4.4: EECF Pose Characteristic Curves for Joint Transducer Parameters

Table 4.2: Joint Voltages and EECF Pose Components for 5 Postures:

Posture	Joint Voltages (volt)			
	Joint 1	Joint 2	Joint 3	Joint 4
1	-11.053	12.369	4.009	-11.583
2	-12.696	9.037	1.493	5.156
3	-13.197	9.964	3.714	-1.898
4	-6.883	13.301	4.122	-28.645
5	-8.793	12.922	3.391	-20.333
6	-9.353	6.637	1.760	11.523
7	-8.917	11.267	1.496	5.202
8	-6.367	12.479	1.369	-4.141
9	-6.034	12.081	0.159	-11.348
10	-6.420	6.843	0.314	-14.182

Posture	EECF Pose Components					
	x (in)	y (in)	z (in)	α (deg)	β (deg)	γ (deg)
1	14.071	19.083	0.177	-69.233	0.000	-90.000
2	6.098	14.220	3.669	175.940	0.000	-90.000
3	5.120	15.465	0.590	-123.644	0.000	-90.000
4	28.576	13.995	0.020	47.600	0.000	-90.000
5	21.851	17.546	1.042	-10.730	0.000	-90.000
6	12.889	13.330	3.326	111.850	0.000	-90.000
7	20.452	16.645	3.695	132.262	0.000	-90.000
8	30.381	14.222	3.873	-170.732	0.000	-90.000
9	30.125	14.500	5.567	-105.410	0.000	-90.000
10	20.507	14.236	5.350	-39.404	0.000	-90.000

In Figure 4.3, some of the characteristic curves for the angular parameters are shown. They are mostly nonlinear relationships as a result of the sine and cosine terms in the coordinate transformation matrices of the kinematic model. Typical characteristic curves for joint transducer parameters are shown in Figure 4.4. All of the EECF pose components are either independent with respect to the variation of the parameters representing the transducer output slopes or repeat themselves after definite intervals. Considering Equation 2.3, it is seen that with θ_{0i} being kept constant, the characteristic curve for k_i repeats itself after every ($360/V_i$) degrees, where V_i represents the voltage of the joint in consideration at the particular posture. This is observed in Figure 4.4c, where the EECF global position repeats itself after every 52.303° interval.

It can also be verified that all the EECF pose components repeat themselves after every 360° interval with respect to the variation of the link Euler angles and the zero point offsets for the revolute joints. This is demonstrated in Figure 4.5, where each of the angular parameters has been varied for 370°, starting at 5° less than the nominal values. The sudden jump of the characteristic curve in Figure 4.5c is attributed to the fact that an angle less than -180° has been converted to a positive angle. For instance, -181° is converted to 179°, and so on. In any case, we have a curve which repeats itself after every 360° variation of the parameter.

Figure 4.5: EECF Pose Characteristic Curves for Joint Transducer Parameters
(360° Variation)

4.4.2 Ill-Conditioning of the System

During the computational runs, the Jacobian matrix was typically found to be moderately ill-conditioned owing to the wide variation of magnitude of its elements. This is shown in Table 4.3, which represents a typical Jacobian matrix (see Equation 3.7). For this particular run, 8 parameters were chosen for optimization, and EECF positioning error data from the first 3 postures of Table 4.2 were considered. In Table 4.3, the parameters are shown at the top of each column. Depending on the posture, the changes in positions of the EECF with respect to the changes in the Euler angles of the most proximal link give rise to elements of the largest magnitudes. The elements representing, for instance, the changes in positions of the EECF with respect to the changes in length parameters are of considerably smaller magnitudes. The ill-conditioning was found to get worse when the Hessian matrix was formed from the Jacobian matrix using Equation 3.9. Table 4.4 shows the Hessian matrix calculated from the Jacobian in Table 4.3. By comparing terms in Tables 4.3 and 4.4 the worsening condition of the Hessian may be observed.

The numerical ill-conditioning of the Hessian is an important issue, as the Hessian must be inverted to get updates, and ill-conditioning introduces numerical error during inversion. Severe ill-conditioning often produces numerical behavior similar to singularity, which makes it impossible to invert a matrix or gives a meaningless numerical solution.

It was found that the ill-conditioning of the Hessian could be avoided to a great extent by judicious choice of angular and linear units. Although the inputs and outputs of angular data in the optimization algorithm were in degrees, all the numerical operations in FORTRAN were done in radians. The Hessian matrix of Table 4.4 also employs radians as its angular unit. An improvement in the conditioning of the Hessian was observed by using degrees as the angular units (as permitted by VAX FORTRAN) instead of using

Table 4.3: Jacobian Matrix (Radians, Inches) from 3 Postures without Parameter Redundancy:

	p_1	p_2	p_3	p_4	p_5	p_6	p_{14}	p_{29}
x	1.00	0.00	0.00	-22.53	-10.55	0.00	-0.33	0.00
y	0.00	1.00	0.00	13.44	0.00	-10.58	0.94	0.00
z	0.00	0.00	1.00	0.00	-13.47	-22.47	0.00	0.00
α	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
β	0.00	0.00	0.00	0.00	0.67	0.74	0.00	-1.00
γ	0.00	0.00	0.00	0.00	0.74	-0.67	0.00	0.00
x	1.00	0.00	0.00	-26.06	-9.51	0.00	-0.05	0.00
y	0.00	1.00	0.00	6.70	0.00	-9.56	1.00	0.00
z	0.00	0.00	1.00	0.00	-6.75	-26.02	0.00	0.00
α	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
β	0.00	0.00	0.00	0.00	0.98	-0.19	0.00	-1.00
γ	0.00	0.00	0.00	0.00	-0.19	-0.98	0.00	0.00
x	1.00	0.00	0.00	-27.58	-10.36	0.00	-0.30	0.00
y	0.00	1.00	0.00	-1.08	0.00	-10.43	0.95	0.00
z	0.00	0.00	1.00	0.00	1.03	-27.56	0.00	0.00
α	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
β	0.00	0.00	0.00	0.00	0.35	-0.93	0.00	-1.00
γ	0.00	0.00	0.00	0.00	-0.93	-0.35	0.00	0.00

Table 4.4: Hessian Matrix (Radians, Inches) from 3 Postures without Parameter Redundancy:

	p_1	p_2	p_3	p_4	p_5	p_6	p_{14}	p_{29}
p_1	3.00	0.00	0.00	-76.16	-30.42	0.00	0.07	0.00
p_2	0.00	3.00	0.00	19.07	0.00	-30.57	2.89	0.00
p_3	0.00	0.00	3.00	0.00	-19.18	-76.05	0.00	0.00
p_4	-76.16	19.07	0.00	2177.32	771.53	-195.04	18.00	0.00
p_5	-30.42	0.00	-19.18	771.53	540.33	449.73	-0.81	-2.01
p_6	0.00	-30.57	-76.05	-195.04	449.73	2256.54	-29.49	0.38
p_{14}	0.07	2.89	0.00	18.00	-0.81	-29.49	3.00	0.00
p_{29}	0.00	0.00	0.00	0.00	-2.01	0.38	0.00	3.00

Note: The Singular Values for this Matrix are 2488.575, 2370.407, 121.603, 5.398, 2.969, 0.149, 0.100, and 0.005.

radians. The resulting Hessian is shown in Table 4.5. The quotient of the largest and the smallest eigenvalues of a symmetric matrix is known as its condition number, and is an indication of the severity of the matrix's ill-conditioning. The larger the condition number, the poorer the conditioning of the matrix. Notice that the condition number decreased from about 446781 (in Table 4.4) to a much smaller value of 61 (in Table 4.5). Even after removing the ill-conditioning caused by unwise choice of physical units, solution of Equation 3.13 was found impossible to obtain due to singularity of the Hessian matrix as discussed further below.

During the computational runs, the *theoretically* singular Hessians, however, were often found to be *numerically nonsingular*, and thus invertible. It was interesting to note that, in many of the cases where the Hessian was nearly singular but invertible, one of the major goals of calibration was still attained, namely, the objective function was reduced considerably. The parameter updates in those cases were strange, however, and the parameters wandered numerically even at solution convergence. The subsequent "optimal" parameters, therefore, did not accurately reflect the physical kinematic characteristics of the manipulator. Even if the sole objective of optimization is that of reducing the end effector positioning error, this is not a reliable approach as it is not possible to predict when the numerical error will be sufficient to make a theoretically singular matrix invertible.

4.5: Hessian Matrix (Degrees, Inches) from 3 Postures without Parameter Redundancy:

	p_1	p_2	p_3	p_4	p_5	p_6	p_{14}	p_{29}
p_1	3.00	0.00	0.00	-1.33	-0.53	0.00	0.07	0.00
p_2	0.00	3.00	0.00	0.34	0.00	-0.53	2.89	0.00
p_3	0.00	0.00	3.00	0.00	-0.34	-1.33	0.00	0.00
p_4	-1.33	0.34	0.00	3.67	0.23	-0.06	0.32	0.00
p_5	-0.53	0.00	-0.34	0.23	3.17	0.14	-0.01	-2.01
p_6	0.00	-0.53	-1.33	-0.06	0.14	3.69	-0.51	0.38
p_{14}	0.07	2.89	0.00	0.32	-0.01	-0.51	3.00	0.00
p_{29}	0.00	0.00	0.00	0.00	-2.01	0.38	0.00	3.00

Note: The Singular Values for this Matrix are 6.274, 5.338, 4.604, 4.402, 1.963, 1.891, 0.945, and 0.102.

4.4.3 Parameter Redundancy

By calculating the singular values of the full Hessian matrix (which is of order 38 in the present model), it was found to be severely rank deficient. Mathematically, the presence of zero singular values in the Hessian means that there are linear dependencies among its columns. From a physical standpoint it means that there are more parameters in this kinematic model than are needed to completely describe a change in the EECF pose of the manipulator being observed. These extra parameters will, henceforth, be termed *redundant parameters* and the phenomenon will be referred to as *redundancy*.

In the general case, it is difficult to identify the columns in the Hessian which are linearly dependent. The singular value decomposition only yields the number of dependency relations among the Hessian columns. Table 4.6 shows a Hessian matrix which was formed from the data of 34 different postures of the A4 manipulator. Eight parameters were chosen to be re-evaluated (optimized). By singular value decomposition, the matrix is shown to have one zero singular value, which implies that there is one linear dependency among its columns. For a general manipulator, the parameters tend to be highly interactive in behavior, and in a typical case one parameter can depend on many other parameters, making the physical interpretation of the dependency relation extremely complicated.

There is, however, a special case of redundancy which is easy to detect and interpret. For the purposes of this work, that particular type of redundancy will be termed *identical redundancy*. In this case, the respective derivatives of all the EECF pose components with respect to the identically redundant parameters will be of equal magnitude regardless of the posture. This results in two or more identical columns in the Jacobian, and consequently in the Hessian. Table 4.7 shows this special case of redundancy. Parameters p_3 and p_9 are identically redundant, and they give rise to two columns whose corresponding elements are

Table 4.6: Hessian Matrix from 34 Postures with One Redundancy:

	p_3	p_4	p_5	p_6	p_7	p_8	p_{10}	p_{13}
p_3	34.00	0.00	-1.37	-10.66	0.00	0.00	0.00	0.00
p_4	0.00	38.53	1.42	-0.18	10.25	-5.68	-35.69	8.18
p_5	-1.37	1.42	35.59	0.38	3.08	-2.47	-0.57	2.73
p_6	-10.66	-0.18	0.38	38.27	-2.46	-3.07	-0.49	2.39
p_7	0.00	10.25	3.08	-2.46	34.00	0.00	-0.91	4.28
p_8	0.00	-5.68	-2.47	-3.07	0.00	34.00	5.68	-27.51
p_{10}	0.00	-35.69	-0.57	-0.49	-0.91	5.68	35.43	-7.00
p_{13}	0.00	8.18	2.73	2.39	4.28	-27.51	-7.00	34.00

Note: The Singular Values for this Matrix are 83.406, 54.315, 46.259, 37.513, 31.466, 24.735, 6.142, and 0.000.

Table 4.7: Hessian Matrix from 34 Postures with One Identical Redundancy:

	p_1	p_2	p_3	p_4	p_5	p_9	p_{11}	p_{12}
p_1	34.00	0.00	0.00	-10.66	-4.55	0.00	-3.07	-2.47
p_2	0.00	34.00	0.00	1.37	0.00	0.00	2.46	-3.07
p_3	0.00	0.00	34.00	0.00	-1.37	-34.00	5.68	-0.98
p_4	-10.66	1.37	0.00	38.53	1.42	0.00	1.36	0.75
p_5	-4.55	0.00	-1.37	1.42	35.59	1.37	23.28	18.93
p_9	0.00	0.00	-34.00	0.00	1.37	34.00	-5.68	0.91
p_{11}	-3.07	2.46	5.68	1.36	23.28	-5.68	35.64	-0.06
p_{12}	-2.47	-3.07	-0.91	0.75	18.93	0.91	-0.06	35.08

Note: The Singular Values for this Matrix are 71.005, 65.746, 45.86, 37.938, 30.283, 24.997, 5.032, and 0.000.

of equal magnitude (although opposite in sign in this particular case). This is a case of direct linear dependency and consequently there is one zero singular value for this matrix, as is shown in the table. Identical redundancy is easy to detect — a visual inspection of the Hessian is sufficient to tell which of its columns are equal and thus which parameters are redundant.

Most commonly, identical redundancy occurs in a robotic manipulator (and in its corresponding kinematic model) if there are two (or more) joints whose axes are parallel in all postures. In the manipulator studied here, the observed conditions for identical redundancy can be classified as follows:

1. For any two or more parallel joints, the parameters of the related link dimensions along the joint axes (Z-component) are redundant. This condition gives rise to 4 identical redundancies among the 5 parameters p_3, p_9, p_{15}, p_{21}, and p_{27}.

2. For two consecutive parallel joints, the Z-Euler angle α for the link between the parallel joints is redundant with the zero-point offset of the distal joint, if the distal joint is revolute. Thus the parameter pairs of p_4-p_{31}, p_{10}-p_{32}, and p_{22}-p_{34} are identically redundant. If the distal joint is prismatic, its zero-point offset has units of length and it does not match up with the angle α of the preceding frame.

3. For two parallel revolute joints with a prismatic joint in between, another redundancy is obtained if the axis of the prismatic joint coincides with the distal revolute joint. In this case, the Z-Euler angle α for the proximal revolute joint is found to be identically redundant with the zero-point offset of the distal revolute joint. Parameter p_{16} is found to be identically redundant with parameter p_{34} for of this reason.

4. For one or more parallel prismatic joints, additional redundancies result. First of all, the length parameter along the joint axis becomes redundant with the zero-point offset of the prismatic joint. Parameters p_{21} and p_{33} are therefore identically redundant. If the distal joint is prismatic, the parameters related to the length components along the X and Y directions also become identically redundant. Parameter pairs p_{13}-p_{19} and p_{14}-p_{20} are found to be identically redundant for this reason. This does not happen if the distal joint is revolute and the frontal joint is prismatic.

Thus it is found that there are in total 11 identical redundancies in the GE model A4 manipulator. If the same manipulator is modeled with a different kinematic model, the number of identical redundancies may change. The first condition stated above will, however, be satisfied by any kinematic notation (complete or incomplete) which assigns its coordinate axes along the axes of the joints. In the Denavit-Hartenberg model, for instance, the joint offsets for any two consecutive parallel axes will be redundant.

4.4.4 Effect of Parameters on the Objective Function

Considerable insight into the nature of the objective function can be gained by observing its changes with respect to the change in only one kinematic parameter at a time. The curve thus obtained will represent the intersection of the multivariable objective function surface with a plane along the axis representing that particular parameter. Following the reasoning described in section 3.2, a curve obtained in such a way is expected to be a parabola, and the magnitude of the objective function calculated at the optimal values of the parameters is located at the vertex of the parabola. In the present case, with synthesized data, this minimum is zero.

Figures 4.6 to 4.10 are all drawn considering the aggregate sum-of-squares errors from ten postures, which are shown in Table 4.2. Figure 4.6 shows the characteristic curves for link length parameters. The parameter value is varied for 10 units on both sides of the optimal value. The curves are smooth and are of perfectly parabolic shape. The minima for these curves are at the optimal values for these parameters, as seen in the figures. As far as these parameters are concerned, the optimization algorithm will have no difficulty in reaching the minimum value in a very few iterations.

Figures 4.7 shows typical characteristic curves for link Euler angles. The parameters are varied for 370°, starting at 5° less than their nominal values. The curves are of a parabolic nature and repeat themselves after every 360° interval of variation. Also, the curves are symmetrical in nature about the nominal parameter values. The repetition of these characteristic curves confirms the fact that after every 360° rotation of these parameters, the manipulator configuration is repeated. Figure 4.8 shows additional characteristic curves for link Euler angles. They are also smooth, symmetrical about nominal values, and self-

Figure 4.6: Sum-of-Squares Characteristic Curves for Link Lengths
(p_1, p_3, p_8, p_{26})

Figure 4.7: Sum-of-Squares Characteristic Curves for Link Euler Angles
(p_4, p_{16}, p_{28}, p_{30})

Figure 4.8: Sum-of-Squares Characteristic Curves for Link Euler Angles (p_5, p_{12}, p_{17})

repeating after every 360°. However, these curves are clearly not parabolic. The reason for their deviation from parabolic shape is not yet clearly known.

As shown in Figure 4.9, the characteristic curves for the parameters representing the joint transducer zero point offsets are essentially of the same nature as those of the link length and link Euler angle parameters. The curve corresponding to p_{33}, which is related to a prismatic joint, is similar to that of a link length parameter and does not repeat itself, as seen in Figure 4.9c.

As explained below, the characteristic curves for the transducer output slopes (revolute joints) are the most complex and interesting in the sense that they do not have a smooth parabolic shape at wide variations from the optimal value. These curves are shown in Figures 4.10a, b, c, and d. The characteristics of the transducer output slope of the 4th joint (p_{38}) is studied in a greater detail in Figure 4.11. For two different postures of the 4th joint (posture #1 and posture #2 in Table 4.2), the variation of the objective function with respect to parameter 38 is shown in Figures 4.11a and b. The joint voltages are -11.583 V and 5.156 V, and the curves repeats themselves after every 31.08° and 69.819°, respectively. These two curves have only one minimum in common, which corresponds to the optimal value. If data from two different postures are taken simultaneously and plotted, Figure 4.11c is obtained. In this case the unique minimum is clearly seen. Finally, Figure 4.11d shows the characteristic curve for 5 simultaneous postures, and it is seen to be highly non-smooth in nature.

Figure 4.9: Sum-of-Squares Characteristic Curves for Joint Zero-Point Offsets (p_{31}, p_{32}, p_{33}, p_{34})

Figure 4.10: Sum-of-Squares Characteristic Curves for Joint Transducer Output Slopes
(p_{35}, p_{36}, p_{37}, p_{38})

Figure 4.11: Detailed Sum-of-Squares Characteristic Curves for p_{38}

The lack in smoothness in the ϕ characteristics for transducer output slopes is important, as it creates a number of valleys at some distance from from the optimal value. Mathematically, this function cannot be called a "convex function." This is dangerous, because if the initial estimate of such a parameter is far away from the optimal value, or if one of the valleys happens to be near to the optimal value, there is a possibility that the optimization algorithm will get stuck in one of those local minima and will fail to converge to the global minimum. Some such cases have been encountered where the magnitude of the gradient converges to a small value, but the objective function remains relatively large. Most of the algorithms available will fail to deal effectively with such a situation. Fortunately, in most cases, the initial guesses of the parameter values (which, in the present work, are the same as the design values) are sufficiently close to the corresponding optimal values and the possibility of the termination of the optimization algorithm before reaching the actual minimum is relatively rare.

4.4.5 Analysis of Solution of Levenberg-Marquardt Algorithm

It has been experimentally verified that the Levenberg-Marquardt algorithm does not preserve the mathematical/physical parameter relationships if the kinematic model contains redundant parameters. While the updated parameters serve to increase end effector positioning accuracy, they do not, in general, correspond to actual dimensions in the case of the redundant parameters. If the minimum of the objective function is zero, the *algebraic sums* of different groups of redundant parameters in the optimal set are found to be the same as they are in the physical set. Otherwise, the algebraic sums, rather than the individual parameters, are found to converge towards a certain value corresponding to the minimum of the objective function. During optimization, these parameters merely readjust themselves so as to produce the unique algebraic sum.

As an example, it has been found that the parameters p_3, p_9, p_{15}, p_{21}, p_{27} and p_{33} of the GE A4 form an identically redundant set. It has also been found that during optimization, the relationship that is preserved is

$$p_3 - [p_9 + p_{15} + p_{21} + p_{27} + p_{33}] = 5.79 \quad . \qquad (4.1)$$

The optimal values for these parameters are found from Table 4.1, and they also satisfy the above equation.

In Table 4.8 results from 5 separate runs are tabulated. Attention is focussed on 8 parameters which are listed at the top of the table. For a particular run, the initial guesses are listed in the first row and corresponding optimal values reached are listed in the second row. The optimal values correspond to the objective function when it reaches its minimum (zero in this case), and they are obtained as the output of the Levenberg-Marquardt algorithm. It is observed that, in each of these cases, the parameters which do not belong to the identically redundant set converge back to their optimal values. This is the proof of correct performance of the optimization scheme. The parameters belonging to the redundant set do not, however, converge to their individual optimal values, but rather, they always satisfy Equation 4.1. The data of Table 4.8 also demonstrates the dependence of optimal values on the initial guesses of identically redundant parameters. Thus, if a certain parameter in a group of redundant parameters undergoes some physical change, in the optimal set that change might not be attributed to that particular parameter, but possibly to all the parameters in that group.

Table 4.8: Typical Convergence Cases for Parameters with an Identical Redundant Set:

p_1	p_3	p_9	p_{10}	p_{15}	p_{21}	p_{27}	p_{33}
Run 1							
14.08	10.00	-0.40	2.00	0.80	1.00	3.00	0.35
15.08	10.09	-0.49	0.00	0.71	0.91	2.91	0.26
Run 2							
14.08	12.00	-0.60	2.00	0.40	1.20	4.00	0.45
15.08	11.87	-0.47	0.00	0.53	1.33	4.12	0.57
Run 3							
17.08	9.00	0.10	2.00	-0.20	0.30	2.80	0.50
15.08	8.97	0.13	0.00	-0.17	0.33	2.83	0.08
Run 4							
17.08	13.00	0.60	2.00	-0.40	1.00	2.50	0.15
15.08	12.44	1.16	0.00	0.16	1.56	3.06	0.71
Run 5							
14.08	12.00	-0.60	3.00	0.40	1.20	4.00	0.45
15.08	11.86	-0.47	0.00	0.52	1.32	4.13	0.57

4.4.6 Analysis of Solution of Singular Value Decomposition

The kinematic model that has been used herein contains more parameters than are actually needed to describe the kinematic relationship between EECF and global frame. Therefore, some of the columns of the system Jacobian matrix are linearly dependent. This results in an infinite number of valid solutions to Equation 3.16. The solution vector with minimum magnitude can be chosen as the unique solution. The imposed condition of minimum magnitude is a purely mathematical one and does not necessarily have any relationship with the physical dimensions of the system concerned. In other words, the method of singular value decomposition is not useful in preserving mathematical/physical parameter relationships.

This technique of solving a singular system is, however, extremely robust. Even if it fails to satisfy the particular need of this work, the method is strongly recommended for those parameter estimation processes where the reduction of the objective function is more important than obtaining a physically meaningful optimal set.

4.4.7 Extent of Redundancy

It was previously mentioned that the full Hessian matrix was found to have a number of zero singular values. This number is equal to the number of extra parameters which are included in the kinematic model but which are not needed to describe the pose of the EECF. In other words, this number indicates the extent to which the chosen model is redundant.

In order get a systematic feeling for redundancy in kinematic models, the mathematical model of a two link 1-DOF manipulator was analyzed. According to the model used, the 1-DOF manipulator needed (6 x 2 + 2)=14 parameters for its description. The parameter values were not kept fixed to the respective values of the original model but were changed arbitrarily in different runs. In each run, the Hessian was constructed and the corresponding singular values were calculated.

Regarding the simulation runs, it has been observed that, in a numerical calculation of singular values, it often becomes very difficult to decide which of the singular values are zero and which are not. A common way is to look for distinct gap in the magnitudes of the singular values. In many cases, it is up to the user to decide a threshold magnitude. A good knowledge of the numerical features of the system and experience with similar problems are very important and useful in making this decision.

The 1-DOF manipulator model was extended up to a 5-DOF manipulator by increasing 1-DOF in each step. Each of the models was tried with widely varying link dimensions, link Euler angles, and joint electrical parameters. The sequence and types of joints were also varied arbitrarily. Eventually it became clear that the number of zero singular values was following a definite pattern.

If the number of zero singular values is subtracted from the total number of parameters in the Hessian matrix, we get the number of linearly independent columns in it. This is the *maximum number of independent parameters* that is needed to completely describe the kinematic features of the manipulator. It was found that this number is a function of only the number and types of joints the manipulator has. If N is the number of independent kinematic parameters, the function can be expressed as

$$N = 5n_r + 3n_p + 6 \quad , \tag{4.2}$$

where n_r and n_p are the number of revolute joints and the number of prismatic joints, respectively. It is interesting to note that N does not depend on the shape or size of the links.

It was also found that each of the joint transducers contribute one independent parameter regardless of the joint type. Therefore, if the electrical parameters for the joint transducers are not optimized, the maximum number of independent parameters will be given by the expression

$$n = 4n_r + 2n_p + 6 . \tag{4.3}$$

The above equation was first formulated from an entirely different approach by Everett, et al. (1987) as a condition of "completeness" of a kinematic model. It was shown that any model, in which the number of kinematic parameters is smaller than that shown in Equation 4.3 could not describe the manipulator completely. This result is also supported by Everett and Suryohadiprojo (1988) in yet another analytical approach.

Another way of verifying Equation 4.3 is by considering the kinematic features of the joints. Denavit and Hartenberg (1955) discussed the minimum number of independent parameters that is required to uniquely specify the joint axes. A prismatic joint is characterized by a vector direction in space with no location specified. Only two parameters are needed to describe this. A revolute joint, on the other hand, is characterized by the location and direction of a vector in space, and consequently needs four parameters to describe it. The global coordinate frame is not constrained in any way with regard to the most proximal local frame, and, therefore, needs a complete set of six parameters. To reduce or remove redundancy, then the number of parameters in the kinematic model of a manipulator must satisfy the condition

$$n \leq 4n_r + 2n_p + 6 . \tag{4.4}$$

It is to be understood that the above condition is not a sufficient condition regarding the absence of redundancy. In other words, the condition does not guarantee that there will be no redundancy in the kinematic model. For instance, if a manipulator has parallel joints, the length components along the joint axes will be identically redundant regardless of whether the total number of parameters satisfies the Equation 4.4. As an example, at the design values of the parameters (as shown in Table 4.1), p_3 and p_9 are identically redundant. If, the parameter p_5 is perturbed from its optimal value of 180°, the global coordinate frame and the frame at the first joint no longer remain parallel. Consequently, p_3 and p_5 are no longer identically redundant and the number of non-zero singular values increases by one.

In Section 2.3, it was concluded that six parameters per link and two or more parameters per joint (depending on linear or nonlinear encoder characteristics) are the minimum number of independent parameters which can account for all the geometric errors in the manipulator and thus preserve an understandable mathematical/physical parameter relationship during optimization. However, we see that in actuality, a lesser number of parameters is sufficient to describe the pose of the EECF in space. Thus a contradiction is reached. On one hand, a proper description of the links for a comprehensive calibration results in a singular system which is impossible to solve while preserving the separate identities of all the parameters. On the other hand, if a non-singular system with just the necessary number of parameters (as defined by Equation 4.3) is obtained, it will be very difficult, if not impossible, to relate a change in one of the optimized parameters to a change in the physical characteristic of a link or in the electrical characteristic of a joint encoder.

4.4.8 Effect of Data Collection at Multiple Sites

It is known that, to improve positional and orientational accuracy of a manipulator, calibration has to be done on the basis of data collected at the EECF. It is implicitly understood at this point that only the final accuracy of the EECF is desirable rather than the accuracy of each and every part of the manipulator body. In other words, from the accuracy standpoint, it is not very important for the i^{th} joint, for instance, to be at the point in space where it is expected to be, as long as the EECF is at the expected position and attitude. In order to improve accuracy of a certain point on the manipulator body, it is necessary that data be collected from that particular point for the estimation of the necessary parameters.

It was proposed by Bosnik (1986) that data collection at several different points on the manipulator body might help in improving the conditioning of the system as far as the evaluation of Equation 3.13 is concerned. This proposition was based on the observation that by collecting data at several well-chosen points on the manipulator, it is possible to make the Jacobian matrix appear to be better conditioned in that it will have no identically equal columns. Also, it intuitively appears that collection of more data is equivalent to the addition of more information to the system model, thereby tending to reduce redundancy. However, it was found that this effort was not successful in producing a non-singular system matrix. The procedure and results are explained below.

As shown in Figure 4.12, three different sites were chosen on the manipulator body, in addition to the site at the tip of the end effector, which became the fourth site. In each posture, positional and orientational data of each site was measured externally. In order to calculate the residuals at a particular site, these externally measured pose components are to

Table 4.9: Typical Jacobian Matrix from One Posture in a Multiple Sites per Posture Calibration Scheme:

Figure 4.12: Location of Sites on the Manipulator Body

be compared with the respective pose components predicted by the internal control algorithm of the manipulator. Due to the fact that exact pose of none of the coordinate frames attached to the sites are available, six unknown parameters are to be added for each new site. These parameters typically relate the position and orientation of a site with respect to the joint frame immediately preceding it. As described at the end of Section 4.3, it is not required to optimize these parameters from a standpoint of improving EECF positioning accuracy. They are included in the optimization scheme to test their usefulness in reducing the redundancy of the system.

Table 4.9 shows a typical Jacobian matrix for the above-mentioned arrangement. In a single posture, data were collected from all four sites on the manipulator, thereby producing 24 rows. Each site except the last produced six additional parameters, thus increasing the number of columns to 56 from of 38. Parameters of link #5 are the same as the parameters of site #4. In the figure, each of the blocks represents a 6×6 submatrix. Some of the submatrices contain only zero elements, and these are shown in the figure by a single zero inside the respective block. This pattern is repeated in each posture.

The important observation regarding Table 4.9 is that there are no identically equal columns in the Jacobian. For instance, in Table 4.7 it was found that the parameters p_3 and p_9 gave rise to equal columns. Table 4.9 shows that multiple site data collection can reduce identically equal columns in the Jacobian.

From Table 4.9, it is possible to see that there can be very few identical columns corresponding to the link parameters. For instance, it is known that the parameters p_3, p_9, p_{15}, and p_{21} give rise to identical columns in the Jacobian matrix. Table 4.9 confirms that multiple point data collection removes these identical columns. Consequently, it can be expected that there will be a reduction in the number of redundancies. This, however, does not happen in practice.

During computational runs it was observed that the number of redundancies in a manipulator remains unchanged even after utilizing positional and orientational data from additional sites. This failure can be viewed most simply by realizing that each site adds six equations to the system and as well as six unknowns to the system. The rank of the system therefore remains unaltered. By calculating the singular values for different Hessians in a similar way, as described in the previous section, it was found that the number of independent parameters N_s in the case of a multiple sites per posture calibration can be calculated as

$$N_s = 5n_r + 3n_p + 6n_s \quad , \tag{4.5}$$

where n_s is the number of sites on the manipulator body including the site at the end effector tip. The above expression shows clearly that the implementation of multiple sites per posture data collection in the calibration algorithm does not help in reducing the number of redundancies.

4.5 Chapter Summary

This chapter describes the computational analysis done under the scope of the present work, including the following :

1. A graphical approach was proposed for obtaining the relationships between the EECF pose components and the kinematic parameters .

2. It was shown that the Jacobian matrix and especially the Hessian matrix are typically ill-conditioned. An effective way of reducing the ill-conditioning was suggested.

3. It was observed that there are more parameters in the kinematic model than are actually needed to completely describe the EECF pose. This phenomenon was termed *redundancy* and was shown to be responsible for creating singularities in the system matrices.

4. A graphical approach was shown to be very useful for depicting the relationship between the objective function and the kinematic parameters of the model. This approach also helps in visualizing the multi-dimensional surface which is a characteristic of the objective function.

5. The solution techniques of the Levenberg-Marquardt algorithm and the singular value decomposition were shown not to be very effective in preserving the physical/mathematical parameter relationship.

6. The number of independent parameters needed to completely describe the EECF pose was found to be a function of the number and the types of joints a manipulator has. This set of parameters was found to be inadequate to describe the kinematic

features of a manipulator and, therefore, is not suitable for a simultaneous optimization scheme.

In summary, therefore, it was observed that the inclusion of all the parameters for the preservation of the physical/mathematical parameter relationship and obtaining a non-singular system matrix are contradictory demands and cannot be satisfied simultaneously. Moreover, for certain manipulators it may be impossible to get a non-singular matrix even with the minimal number of parameters, as defined by Equation 4.3. The collection of data at multiple sites per posture was found not to be useful in reducing redundancy.

Chapter 5

SUMMARY, CONCLUSIONS, AND FUTURE WORK

5.1 Summary

Calibration has been found by many investigators to be a practical solution to the problem of increasing the accuracy of the EECF pose of a robotic manipulator. By an appropriate choice of kinematic model and calibration algorithm, the optimized kinematic parameters can be made to represent closely the actual physical dimensions of the manipulator in many cases. Such information is helpful in identifying sources of error, implementing predictive maintenance schemes, and hence, in predicting component damage or wear before it might be discernible by other means. The preservation of these mathematical/physical parameter relationships has been a primary goal of the current work.

An appropriate choice of kinematic model and coordinate frame locations on the manipulator structure has been found to affect both success of the calibration algorithm and preservation of the mathematical/physical parameter relationships. A minimum of six parameters per link and two parameters per joint (considering linear joint transducer characteristics) is necessary for full description of the manipulator for calibration purposes. The model proposed by Sheth and Uicker (1971) is useful in calibration, although a slightly different model (Bosnik, 1986) was used in the current work. The various kinematic model/calibration algorithm combinations were evaluated via a numerical implementation of the Levenberg-Marquardt nonlinear least squares optimization algorithm, utilizing both experimentally collected and synthesized data.

The existence of redundant kinematic parameters in the manipulator kinematic model complicates the problem of preserving the mathematical/physical parameter relationships considerably. The redundant parameters interact among themselves during optimization in such a way that the final result depends on the initial guesses and also in such a way so as to satisfy a condition which does not have any direct correlation with the physical features of the manipulator.

The straightforward method of collecting data at different sites on the robot body was attempted with the intention of reducing redundancy. This attempt was not successful, as the extra information provided by additional data was nullified by the extra number of parameters incorporated into the model. The number of redundant parameters remained unchanged.

5.2 Conclusions

The primary goal of this work was to explore the possibility of establishing a relationship between the physical parameters of a manipulator and the corresponding optimal mathematical parameters obtained as a result of calibration. This knowledge has potential application in tracking the wear and damage of various parts of the manipulator body.

The comparison of the initial parameters and the changed parameters is physically meaningful only if the manipulator possesses a clearly discernible mathematical/physical parameter relationship. It was shown that, in a useful kinematic model, there should be six parameters per link and two parameters per joint for purposes of calibration.

During the computational runs it became evident that the EECF pose changes can be completely described with fewer independent parameters than the number required by the "useful" model described above. Thus the original set of parameters describing the kinematic features include "useful" redundant parameters, making the system singular. It therefore becomes impossible to get a meaningful solution as far as the preservation of physical/mathematical parameter relationship is important. In this work it also became apparent that it is impossible to get rid of redundancy in a complete calibration procedure which requires simultaneous optimization of all the parameters and uses a kinematic model that includes a full description of each individual link and joint.

One way to reduce the severity of rank deficiency of a system is to optimize fewer parameters. The exclusion of some of the parameters from the optimization scheme will yield useful results only if their values are known to a high degree of accuracy. The number of redundant parameters in the system may be reduced or eliminated by using such a technique, although the opportunity to obtain meaningful mathematical/physical parameter relationships is concomitantly reduced. Some guidelines are available; it is generally known, for example, that a major portion of the total kinematic error in a manipulator is typically due to inaccurate knowledge of the joint transducer zero-point offset values (Bosnik, 1986). Judd and Knasinski (1987) observed that inaccurate knowledge of these parameters might contribute up to 95% of the RMS error in the EECF pose. Therefore, zero-point offsets should generally not be removed from the optimized parameter set.

The parameter identification method suggested by Stone (1987) is interesting from the standpoint of its effects on parameter redundancy. Stone's technique does not rely on the link dimensions supplied by the manufacturer for initial parameter guesses; rather, parameters are deduced from performance data of the manipulator. The technique employs a variation of the multiple site data collection scheme described above and performs separate local optimization at each joint. The set of parameters suggested by Stone exceeds the number of parameters that can be had in a system without redundancy.

5.3 Future Work

The research work presented here can be extended towards improving the calibration procedure of a manipulator. Some salient points are:

1. As mentioned in Chapter 2, our present knowledge about the behavior of the non-geometric errors is far from satisfactory at present. These errors are caused by factors such as link compliance, gear backlash, gear train compliance, motor-bearing wobble, etc., and may contribute significantly to the end effector positioning error. A systematic way of parameterize these errors will represent an important step towards a more complete static calibration.

2. The elastic effects which occur during the operation of manipulators are another class of factors which contribute to the kinematic error. There is already a trend in the industry towards manufacturing manipulators which are lighter in weight and/or faster in operation. Elastic effects will play a significant role in the accuracy of this new generation robots and an effort towards accurately modeling these effects will be valuable.

3. The dynamic and vibrational factors also contribute to the kinematic errors of a robot during its operation. Consequently, dynamic calibration will considerably improve the accuracy of the robot from this standpoint. The dynamic factors cannot be accounted for in a static calibration scheme and therefore new methods need to be developed and/or refined.

In all likelihood, an integrated approach towards calibration will eventually emerge. This approach will attempt to minimize the observed errors in a manipulator by suitably adjusting all the static, elastic, and dynamic parameters simultaneously. A final scheme might be devised in which a robot will automatically "sense" the need for calibration (when its observed error exceeds some threshold value) and will run the calibration algorithm without the input of a human operator. Thus the internal control parameters will be continuously updated and will maintain the manipulator performance at an optimum level. Such an approach would provide the greatest gain in performance which appear to be attainable through robot calibration.

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