Generation of Energy Optimal Complete Gait Cycles for Biped Robots

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Abstract

In this paper we address the problem of energy-optimal gait generation for biped robots. Using a simplified robot dynamics that ignores the effects of centripetal forces, we obtain unconstrained optimal trajectories generated by piecewise constant inputs. We study a complete gait cycle comprising single support, double support and the transition phases. The energy optimal gaits for different step lengths and velocities are compared with natural human gait.

1 Introduction

Recently, many studies have been devoted to locomotion, path planning and control of biped robots [3]-[6]. The main motivations for using walking robots rather than more conventional wheeled robots are their versatility in moving in unstructured and rough terrain and for their obstacle avoidance capabilities. In particular, bipeds robots, under suitable mechanical design, are potentially capable of producing gaits involving very little input energy (apart from the restitution energy needed to compensate for the losses due to friction and contact).

This last point has raised a lot of interest since the generation of low-energy trajectories for biped robots remains an open and non-trivial issue [3] [2] [8] [1]. From a practical point of view it seems reasonable to search for a trajectory that fulfills a certain objective in terms of the gait parameters (such as the walking velocity, the step length and the step frequency) while minimizing the input energy needed to produce such a gait.

The problem of determining whether or not these so called natural gaits exist is complex and only a few partial analytical results are available up to now, see [5]. This problem is equivalent to finding the joint velocity values before and after the swing and double support phases such that the desired gait cycle is reproduced. To this problem may be derived closed-form solutions provided that explicit integration of the support phase equations can be performed and that a reasonable model for contact losses is available. This is feasible only for simple systems such as the monopod [6], or if linearized model is considered.

The energy-optimal trajectories for highly nonlinear equations of a complex robot may be found only numerically, and in general, they will be sub-optimal. Earlier investigations for numerical solutions approximated the joint trajectories to time-polynomials [3] [2], Fourier expansions [1], or a combination of both [8]. An exhaustive treatment of the application of the optimal programming to human locomotion is given in [4], where penalty functions are used to minimize the total mechanical work done. This technique is now superseded by new numerical optimization algorithms.

In this paper we propose an alternative method for energy optimal gait generation. The proposed approach searches for unconstrained trajectories (no particular time or frequency base function are chosen) generated by piecewise constant inputs. [1] A numerical study presented in [7] has shown that, for an equivalent amount of computational burden, this method provides motions with a lower input energy compared to polynomial or Fourier extensions. The solution is then found numerically after transforming the dynamical problem into a static one and solving it via a standard direct shooting optimization algorithm.

[1] Although we explicitly mention that our control input is piecewise constant, we need to remember that the numerical optimization always necessitates a discretization of the system equations resulting in piecewise constant control inputs.

2 Problem Formulation

A complete human gait cycle may be divided into two phases: the single support phase or the swing phase (one foot on the ground and the other foot swinging) and the double support phase (both feet on the ground). The transition from the single support to the double support phase, also called the *contact phase*, is associated with the heel of the front foot touching with the ground. The transition from the double support to the single support phase, also called the *take-off phase*, is caused when the toe of the rear foot leaves the ground. The dynamic equations of a robot consisting of all the described phases is composed of ordinary differential equations for the support phases and algebraic equations for the transition phases. Moreover, the robot's kinematic topology changes from the single support to the double support phase complicating further the differential equations.

It is not an easy task to choose a biped kinematics that captures the essence of the anthropomorphic gait while keeping the model reasonably simple to allow intuitive insights about its behavior. Admitting the fact that the simplifications may sacrifice some of the subtleties of human motion, we have converged upon a planar four degrees-of-freedom (DOF) biped mechanism as shown in Figure 1. For our study the following planar biped robot model will be used. The trunk mass m is located at the hip. a_1 and a_2 are the lengths of the shank and the thigh, respectively. Their masses are m_1 and m_2 respectively. See Figure 1 for a sketch of the model. We have assumed that in this model the trunk will be upright during the walk. This seems reasonable because the trunk's maximal excursion from the vertical axis is about 20mm at the pelvis point, see [8]. The foot of the swing leg is considered massless thereby obviating motor in the corresponding joint.

2.1 The Gait Phases

Mimicking the human gait phases, the dynamics of the biped robot can be decomposed into four different phases describes bellow.

• **Single Support Phase.** The dynamic equations, of this phase, derived by means of the familiar Euler-Lagrange formulation. Biped robot is modeled as a rotational joint open-chain manipulator in this phase.

$$\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}_{ss} \quad 0 < t < T_1 \qquad (1)$$

where $\boldsymbol{q} \in R^4$ describes the generalized coordinates, $\boldsymbol{H}(\boldsymbol{q})$ is the inertia matrix, $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the matrix of centripetal acceleration and Coriolis terms, $\boldsymbol{g}(\boldsymbol{q})$ is the gravity vector, $\boldsymbol{u}_{ss}$ is the input torque vector during single support phase, and T_1 is the time duration of the swing phase.

• **Contact Phase.** During this phase, the robot configuration remains unchanged, $\boldsymbol{q} = const.$, while there is a discrete change in the joint velocities such that the swing foot stays on the ground after contact. The contact phase is assumed to be instantaneous and inelastic, and without sliping. Centripetal torques are assumed to be smaller than the impulsive forces and are neglected. Therefore, the dynamic equations can be integrated in order to establish the relationship between joint velocities just before and just after the impact:

$$\boldsymbol{H}(\boldsymbol{q})(\dot{\boldsymbol{q}}^+ - \dot{\boldsymbol{q}}^-) = \boldsymbol{I}_{cont} \quad t = T_1 \qquad (2)$$

where $\dot{\boldsymbol{q}}^-$ and $\dot{\boldsymbol{q}}^+$ are respectively the joint velocity just before and just after the contact and $\boldsymbol{I}_{cont}$ is the impulse of the impact force which is active during the contact phase.

• **Double Support Phase.** As in the single support phase, the dynamic equations of the double support phase are also derived from the Euler-Lagrange formulation. The fact that the robot foot stays on the ground adds two supplementary constraints of the form x_f = const, $y_f = 0$, thereby reducing the admissible set of joint coordinates. The constraints are expressed as $\Phi(\boldsymbol{q}) = 0$, and the use of Lagrange multipliers, $\boldsymbol{\lambda}$, allow us to write the dynamic equations as,

$$\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}_{ds} + \boldsymbol{J}_{\Phi}^T\boldsymbol{\lambda} \quad T_1 < t < T \qquad (3)$$

where $\boldsymbol{J}_{\Phi} = \frac{\partial \Phi}{\partial \boldsymbol{q}}$ is an $m \times n$ Jacobian matrix, $\boldsymbol{u}_{ds}$ is the input torque vector active during the double support phase, and T, the time of the complete gait cycle.

• **Take-Off Phase.** As in the contact phase, the robot configuration is constant while a change of the support foot reference appears. The joint velocity is discontinuous to bring the robot to the initial state of the following single support phase. This condition must be held to assure a cyclic walk.

$$\boldsymbol{H}(\boldsymbol{q}^+)\dot{\boldsymbol{q}}^+ - \boldsymbol{H}(\boldsymbol{q}^-)\dot{\boldsymbol{q}}^- = \boldsymbol{I}_{top} \quad t = T \qquad (4)$$

$\boldsymbol{q}^-$ is the robot's joint coordinates just before the take-off and is computed with respect to a coordinate frame at the support foot for the previous swing phase whereas $\boldsymbol{q}^+$ is the robot's joint coordinates just after the take-off and is computed with respect to a coordinate frame at the new support foot.

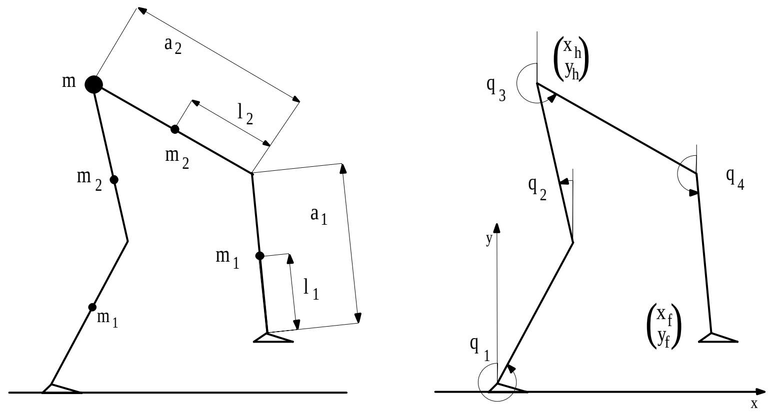

Figure 1: Simplified structure of the 4-DOF biped robot used for our study.

The complete dynamics of the robot is therefore given by the equations (1)-(4). Henceforth in the paper we will use a state-space description of the robot with the state vector $\boldsymbol{x} = [\boldsymbol{x}_1^T, \boldsymbol{x}_2^T]^T$ where $\boldsymbol{x}_1$ is the joint position vector and $\boldsymbol{x}_2$ is the joint velocity vector. In this description the complete dynamics of the robot (equations (1)-(4)) can be rewritten as

$$\left\{ \begin{array}{lcll} \dot{\boldsymbol{x}} & = & f_{ss}(\boldsymbol{x}, \boldsymbol{u}_{ss}) & t \in [0, T_1[\\ \boldsymbol{x}^+ & = & \phi_{cont}(\boldsymbol{x}^-, \boldsymbol{u}_{cont}) & t = T_1 \\ \dot{\boldsymbol{x}} & = & f_{ds}(\boldsymbol{x}, \boldsymbol{u}_{ds}) & t \in]T_1, T[\\ \boldsymbol{x}^+ & = & \phi_{top}(\boldsymbol{x}^-, \boldsymbol{u}_{top}) & t = T \end{array} \right. \qquad (5)$$

where $\boldsymbol{u}_{cont} = \boldsymbol{I}_{cont}/dt$ and dt is the impulsive time of the contact phase, and $\boldsymbol{u}_{top} = \boldsymbol{I}_{top}/dt$ and where the functions f_{ss}, ϕ_{cont}, f_{ds}, ϕ_{top} can be obtained from the equations (1)-(4).

Due to the difficulties in the convergence of the optimization procedures with the complete dynamics, we introduce certain model simplifications. In the first place, our goal is to obtain optimal gaits to understand the walk phenomenon and for this, the simplifications are the follows. The inertia matrix is supposed to be constant and diagonal, and the centripetal accelerations are ignored. This assumption is reinforced by the fact that gear ratios of the D-C actuators are large enough so that coupling and position dependent terms of the inertia matrix can be ignored. The main nonlinearities considered by our model are the gravity terms. Then the dynamic model becomes:

$$\left\{ \begin{array}{lcll} \boldsymbol{H}\ddot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) & = & \boldsymbol{u}_{ss} & t \in [0, T_1[\\ \boldsymbol{H}(\dot{\boldsymbol{q}}^+ - \dot{\boldsymbol{q}}^-) & = & \boldsymbol{u}_{cont} & t = T_1 \\ \boldsymbol{H}\ddot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) & = & \boldsymbol{u}_{ds} + \boldsymbol{J}_\Phi^T \boldsymbol{\lambda} & t \in]T_1, T[\\ \boldsymbol{H}(\boldsymbol{q}^+)\dot{\boldsymbol{q}}^+ - \boldsymbol{H}(\boldsymbol{q}^-)\dot{\boldsymbol{q}}^- & = & \boldsymbol{u}_{top} & t = T \end{array} \right.$$

2.2 The Cost Function

The following cost function is used for our optimal control scheme:

$$\begin{array}{rl} J & = \int_0^{T_1} \boldsymbol{u}_{ss}^T \boldsymbol{u}_{ss}\, dt \; + \; \boldsymbol{I}_{cont}^T \boldsymbol{I}_{cont} \\ & + \int_{T_1}^{T} \boldsymbol{u}_{ds}^T \boldsymbol{u}_{ds}\, dt \; + \; \boldsymbol{I}_{top}^T \boldsymbol{I}_{top} \end{array} \qquad (6)$$

which quantifies the injected energy into the robot during a gait cycle. The injected energy to the robot is a reasonable criterion to minimize, especially for mobile robots that needs to carry their own power source.

The specified boundary conditions for the optimization are the initial and terminal joint angles of the single support phase $\boldsymbol{x}_1(0)$, $\boldsymbol{x}_1(T_1)$, and the final joint angle $\boldsymbol{x}_1(T)$ of the double support phase. We also specify T_1, and T, the time intervals of the swing phase and the total cycle time, respectively. This implicitly imposes an average progression speed for the robot. Please note that the three corresponding velocities $\boldsymbol{x}_2(0)$, $\boldsymbol{x}_2(T_1)$ and $\boldsymbol{x}_2(T)$ are free. This represents an additional degree of freedom giving the possibility for the optimization procedure to characterize minimum-energy trajectories. Let us define the variable $\boldsymbol{u}$ as:

$$\boldsymbol{u} = [\boldsymbol{u}_{ss}^T \; \boldsymbol{u}_{cont}^T \; \boldsymbol{u}_{ds}^T \; \boldsymbol{u}_{top}^T]^T \qquad (7)$$

As a resume the optimal control problem can be states as:

Problem formulation 2.1 *Given the initial and final joint angles of the single support phase* $\boldsymbol{x}_1(0) = \boldsymbol{x}_{1_0}$, $\boldsymbol{x}_1(T_1) = \boldsymbol{x}_{1_{T_1}}$, *and the final joint angles* $\boldsymbol{x}_1(T) = \boldsymbol{x}_{1_T}$ *of the double support phase, and the time intervals* T_1 *and* T, *the problem is to find the optimal sequence* $\boldsymbol{u}^\star(t)$, *minimizing the cost function* J *(Eq 6), such that it steers the system (5) from* $\boldsymbol{x}_{1_0}$ *to* $\boldsymbol{x}_{1_T}$.

3 Optimization

The dynamic optimal control problem is next transformed into a static problem via a discretization as we describe subsequently.

3.1 The Piecewise Constant Method

We assume that the control input $\boldsymbol{u}(t)$ is piecewise constant. Let N_1 be the number of time-intervals during the single support phase and N the total number of intervals during single and double support phases. The input control sequence is composed of the single support control phase sequence, the impact control, the double support phase control sequence and the take-off phase control. We now define $\boldsymbol{U} \in R^{(4\times N+2)}$ as the input control matrix:

$$\boldsymbol{U} = [\,\underbrace{\boldsymbol{u}_0, \boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_{N_1-1}}_{\boldsymbol{U}_{ss}}\,,\ \boldsymbol{u}_{cont}, \underbrace{\boldsymbol{u}_{N_1+1}, \ldots \boldsymbol{u}_{N+1}}_{\boldsymbol{U}_{ds}}\,,\ \boldsymbol{u}_{top}\,]$$

Then, with this assumption, the cost function (6) can be reformulated as:

$$\begin{aligned} \mathcal{C} = \quad & \textstyle\sum_{k=0}^{N_1-1} \boldsymbol{u}(k)^{\mathcal{T}}\boldsymbol{u}(k)\Delta t \ + \ \boldsymbol{u}_{cont}^{\mathcal{T}}\boldsymbol{u}_{cont}\Delta t \\ & + \textstyle\sum_{k=N_1+1}^{N+11} \boldsymbol{u}(k)^{\mathcal{T}}\boldsymbol{u}(k)\Delta t \ + \boldsymbol{u}_{top}^{\mathcal{T}}\boldsymbol{u}_{top}\Delta t \end{aligned} \quad (8)$$

The dynamic problem (2.1) will be transformed to a static one by approximating the derivative operator by the Euler's formula in the state equation during the two continuous time phases of the gait cycle, (1) (3). The two other phases are already discrete time representation and thus, this approximation has no influence on them.

• **Single support phase.** During this phase, the derivative operator $\dot{\boldsymbol{x}}$ is approximated as follows:

$$\dot{\boldsymbol{x}}(k+1) = \frac{\boldsymbol{x}(k+1) - \boldsymbol{x}(k)}{\Delta t} = f_{ss}(\boldsymbol{x}(k), \boldsymbol{u}(k))$$

It is then possible to write the state at the instant $k+1$ as a function of the previous state $\boldsymbol{x}(k)$ and the input control $\boldsymbol{u}(k)$. By induction, the final state $\boldsymbol{x}(N_1)$ of the swing phase can be written as a function of the initial state $\boldsymbol{x}(0)$ and the input control sequence of the single support phase $\boldsymbol{U}_{ss}$:

$$\begin{aligned} \boldsymbol{x}(N_1) &= F_{ss}(\boldsymbol{x}(0), \boldsymbol{u}(0), \cdots, \boldsymbol{u}(N_1-1)) \\ &= F_{ss}(\boldsymbol{x}(0), \boldsymbol{U}_{ss}) \end{aligned} \quad (9)$$

where the operator F_{ss} is defined as:

$$F_{ss} = f_{ss} \circ f_{ss} \cdots \circ f_{ss}(\boldsymbol{x}(0), \boldsymbol{u}(0)) \quad (10)$$

• **Contact Phase.** By considering (2), which is already a discrete time equation, the state at the instant N_1+1 can be uniquely defined from the final state of the single support phase $\boldsymbol{x}(N_1)$, which comes from (9), and the contact control $\boldsymbol{u}_{cont} = \boldsymbol{u}(N_1)$:

$$\boldsymbol{x}(N_1+1) = \phi_{cont}(\boldsymbol{x}(N_1), \boldsymbol{u}(N_1)) \quad (11)$$

• **Double Support Phase.** In the same way as the swing phase, it is possible to express the final state $\boldsymbol{x}(N)$ as a function of the initial state of this phase that is to say $\boldsymbol{x}(N_1+1)$, which is obtained from (11), and the input control sequence of the double support phase $\boldsymbol{U}_{ds}$:

$$\begin{aligned} \boldsymbol{x}(N) &= F_{ds}(\boldsymbol{x}(N_1+1), \boldsymbol{u}(N_1+1), \cdots, \boldsymbol{u}(N)) \\ &= F_{ds}(\boldsymbol{x}(N_1+1), \boldsymbol{U}_{ds}) \end{aligned} \quad (12)$$

where the operator F_{ds} is defined as:

$$F_{ds} = f_{ds} \circ f_{ds} \cdots \circ f_{ds}(\boldsymbol{x}(0), \boldsymbol{u}(0)) \quad (13)$$

• **Take-off Phase.** As in the impact phase, by considering (4), the state at the instant $N+1$ can be uniquely defined from the final state of the DS $\boldsymbol{x}(N)$, which comes from (12), and the link control $\boldsymbol{u}_{top} = \boldsymbol{u}(N)$:

$$\boldsymbol{x}(N+1) = \phi_{top}(\boldsymbol{x}(N), \boldsymbol{u}(N)) \quad (14)$$

The new problem formulation can be stated as the following static problem:

Problem 3.1 *Given the initial and final joint angles $\boldsymbol{x}_1(0)$, $\boldsymbol{x}_1(N_1)$ and $\boldsymbol{x}_1(N)$ and the time intervals T_1 and T, the problem is to find the optimal value for $\boldsymbol{U}^\star(t)$, minimizing the cost function $\mathcal{C}$ (8), such that it steers the system (5) from $\boldsymbol{x}_1(0)$ to $\boldsymbol{x}_1(N)$. Or equivalently:*

$$\left\{ \begin{array}{l} \min\limits_{\boldsymbol{U}} \ \mathcal{C}(\boldsymbol{U}) \\ \\ under \left\{ \begin{array}{lcl} \boldsymbol{x}(N_1) &=& F_{ss}(\boldsymbol{x}(0), \boldsymbol{U}_{ss}) \\ \boldsymbol{x}(N_1+1) &=& \phi_{cont}((\boldsymbol{x}(N_1), \boldsymbol{u}(N_1)) \\ \boldsymbol{x}(N) &=& F_{ds}(\boldsymbol{x}(N_1+1), \boldsymbol{U}_{ds}) \\ \boldsymbol{x}(N+1) &=& \phi_{top}((\boldsymbol{x}(N), \boldsymbol{u}(N)) \end{array} \right. \\ \\ given\ (\boldsymbol{x}_1(0), \boldsymbol{x}_1(N_1), \boldsymbol{x}_1(N), T_1, T, N_1, N) \end{array} \right.$$

In the following section, we describe the results obtained by simulation.

3.2 Optimal Energy Gaits

In Figure 2 we can see an example of optimal gait corresponding a step length 0.4 m and $T = 1$ s. This gait needs of 0.57 $Cal/m/kg$ with a walking velocity equal to 0.4 m/s. The gait of the biped is not similar during a complete gait cycle and during only a single

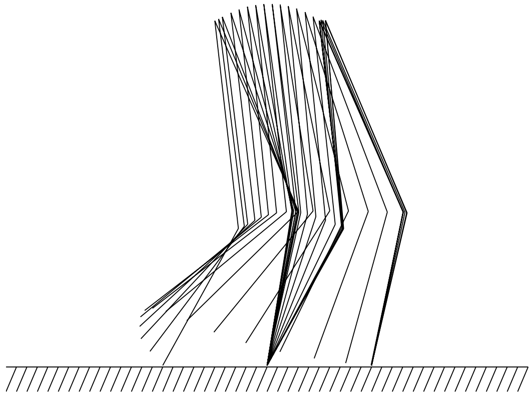

Figure 2: Optimal Gait obtained during a complete gait cycle with a step length $S = 0.4m$ and a time period $T = 1s$.

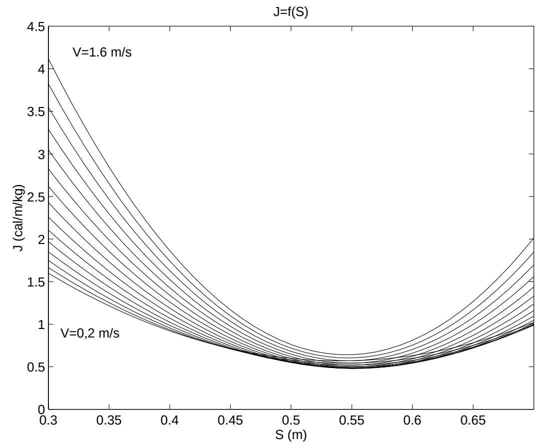

Figure 3: Optimal Criterion as a function of S for different values of V.

support phase [7] which produces ballistic gait. While during a complete gait cycle, the optimization takes all phases in consideration to find a minima, that is the reason why the foot of the swing is more "controlled" during the complete gait cycle in order to not penalize the cost function through the transition phases. The required energy to reach a step is more distribute during the complete cycle.

As the energy optimal gait generation is not sufficiently fast to be computed in real time, the aim of this study is to establish a database of pre-computed optimal gaits.

The walk parameters, the step length S, the time period T and as a consequence the walking velocity V, vary. The resulting cost criterion represents the energy required to perform one meter per kg. Figure 3 shows that each walking desired velocity corresponds to an optimal step length. It can be seen that, for all velocities, smaller steps correspond to higher energy consumption. It is essentially due to the greater number of steps required to cover the same distance and then the greater number of impact and take off phases involved. The main result which comes from this figure, it is that for all velocities, the optimal step length is the same, and it is approximately $0.54m$. That is to say, for each desired velocity, there exist a couple of optimal walking parameters S and T.

Figure 4, shows the cost function versus walking velocity for different step lengths, that for each step length, there is an optimal walking velocity. For human locomotion, this fact is characterized and this velocity is called “natural” walking velocity. If we are not constrained to walk at a certain velocity, this “natural” velocity is always adopted and then we choose to minimize our energy consumption. The profile of such human curve is shown in figure 5.

It is interesting to compare our results with those obtained from the study of normal human gaits. In the biomechanics literature, it is generally accepted that the normal human gait minimizes energy expenditure per unit body weight per distance although the function is rather flat at the minimum and a modification of the gait velocity does not significantly change the energy expenditure. According to [?] a grand average of most of the data available at that time gave a value of optimal walking speed of 80 m/min and at that velocity the human body spends 0.8 calorie of energy per meter of distance traveled per each kilogram of body weight. For the robot model considered in this paper we have approximatively one calorie of energy per meter.

A third thing in human gait is that people, in unforced walk, tends to use step lengths that are proportional to the cadence.

The figure 6 shows the percentage of consumed energy during each phase of a complete gait cycle. Firstly, the energy losses during the impact phase do not appear to be so important. It is due to the objective function which penalize the total amount of energy. Then, the best strategy (approach, procedure, scheme) is not necessarily find a ballistic swing phase if the impact phase losses a large quantity of energy due to the difference between the velocities just before and just after the impact. The optimization scheme tends to control all the swing phase in order to reduce the consumed energy during the impact: it is a compromise. After, it can be noted that the energy losses

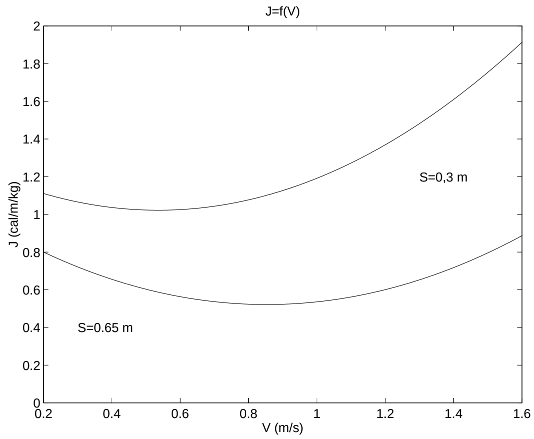

Figure 4: Optimal Criterion as a function of V for different values of S.

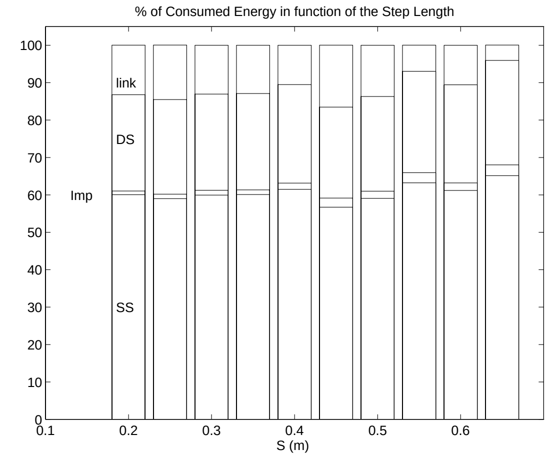

Figure 6: Repartition of Consumed Energy during the four phases.

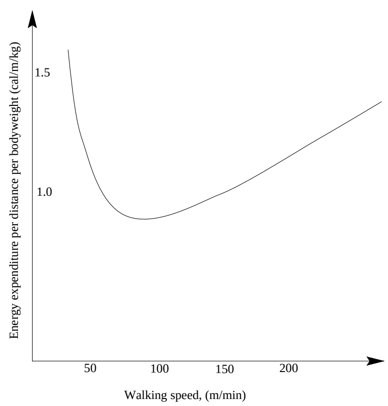

Figure 5: "Natural" speed in human case.

during the impact increase with the step length.

4 Conclusion

This article studies a complete gait cycle including the single support phase, the impact, the double support phases and the link phase. A generator of energy optimal gait is realized and some energizing results are compared to human results.

It is also interesting to study the potential benefits of introducing passive elements in the joint articulations [1], which appear as a natural way enhancing a passive walk.

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