Driver Assist for Backing-Up a Vehicle with a Long-Wheelbase Dual-Axle Trailer

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Backing-up of articulated vehicles poses a difficult challenge even for experienced drivers. While long wheelbase dual-axle trailers provide a benefit of increased capacity over their single-axle counterparts, backing-up of such systems is especially difficult. We propose a control strategy for such systems, introducing concepts of the *hitch control space* and *no-slip curve* derived from no-slip kinematics, allowing backing-up maneuvers to be intuitive to drivers without experience with trailers. Using hitch angle feedback, we show these concepts can be used to stabilize the trailer in back-up motion in the presence of arbitrary driver inputs. The controller is tested in simulation and on a scale model testbed, demonstrating that robust and stable backing-up of such systems can be achieved whilst allowing the driver to maintain full control of the vehicle.

Topics / Vehicle Dynamics, Articulated Vehicles, Reversing, Backing-up Control, Intelligent Vehicle Control

1. INTRODUCTION

The backing-up motion of articulated vehicles naturally exhibits unstable behavior [1], [2]. Such maneuvers often require counterintuitive inputs, making them error-prone, cumbersome, frustrating or worse, dangerous, especially for uninitiated drivers. In this paper we propose an assist algorithm for the backing up of an articulated system, in which the trailer is dual-axle, has a long wheelbase and is equipped with rear axle steering. Existing backup controllers for traditional trailers require partial or full control of the vehicle steering angle [3], [4]. These control algorithms achieve stabilization by driving the articulated system to a target hitch angle via hitch angle feedback, where the target angle is derived using the kinematic condition of no-slip for the articulated system [5].

In [6], the authors make use of rear-wheel steering of the vehicle to accomplish hitch angle stabilization, allowing the front steering angle of the vehicle to remain fully controlled by the driver. Novel concepts in trailer stabilization have also been proposed, such as a laterally moving hitch point [7], [8], however implementation of such method would require extensive modifications to the vehicle hardware. Another method of trailer stabilization using curvature tracking has also been examined, where the path curvature of the trailer is the control target [9]. This technique does not utilize a reference hitch angle, kinematically determining path curvature from the steering angle, however requires a pre-computed path to follow.

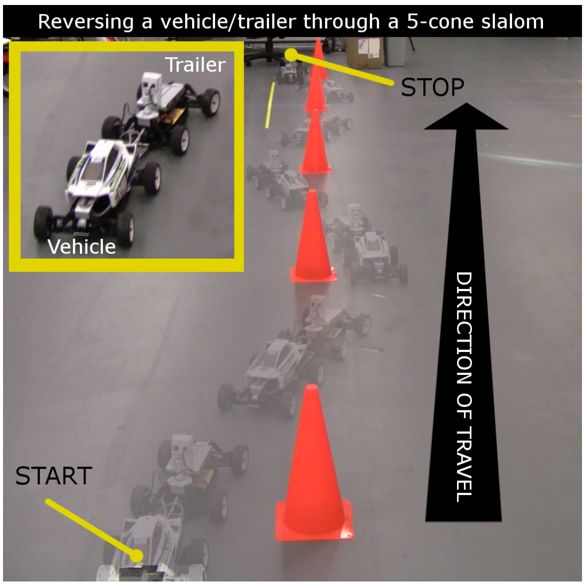

Fig. 1: Progression of vehicle-trailer system through a 5-cone slalom under stable back-up control is shown using superimposed snapshots.

In this paper we propose a method of hitch angle stabilization achieved solely via trailer steering, allowing the driver to be in complete control of the vehicle, and to reverse the articulated system in a way that it behaves similar to a vehicle without any trailer. This allows for all of the necessary control actuators and additional sensors to be located on the trailer. The proposed control strategy makes reversing articulated systems considerably easier for the driver, allowing for complex backing-up maneuvers such as a 90° corner and a 5-cone slalom, the latter shown in Fig. 1. Left uncontrolled, there is a high probability of jack-knife as the desired motion requires the driver to constantly make adjustments to the vehicle steering, indirectly maneuvering the trailer through the obstacles.

2. DYNAMIC MODEL OF AN ARTICULATED VEHICLE-TRAILER SYSTEM

We assume that the vehicle and the trailer, which are connected by a single degree-of-freedom (DoF) un-actuated hitch H (shown in Fig. 2), are free to move about a horizontal planar surface, eliminating roll and pitch dynamics of the coupled system. The hitch allows relative yaw motion between both the vehicle and trailer.

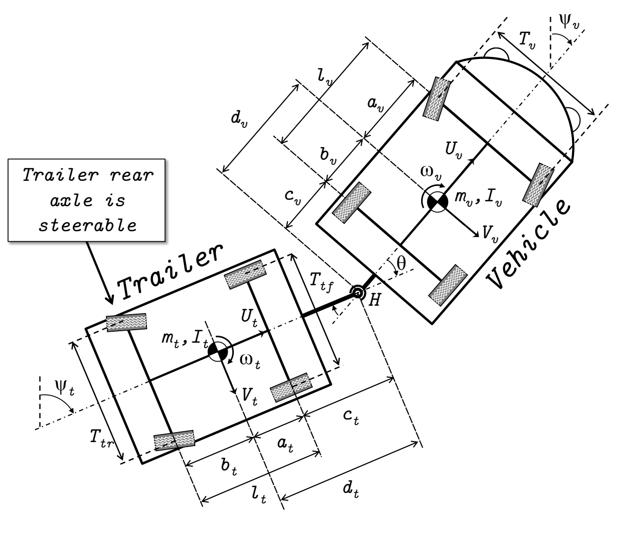

Fig. 2: Vehicle with a long wheelbase dual-axle trailer. Notations and symbols are explained in the text.

The motion of this system is governed by the throttle, brake and steering of the vehicle, the trailer steering and tire/ground contact forces. For the latter we use a simplified Magic Formula tire model [10]. In this system, the vehicle has front steering and the trailer has rear steering. The resulting equations of motion of this planar system are given by:

$$
\begin{aligned}
m_v\left(\dot{U}_v - V_v\omega_v\right) + m_t\left(\dot{U}_t - V_t\omega_t\right)\cos\theta \\
-m_t\left(\dot{V}_t + U_t\omega_t\right)\sin\theta \\
= \sum F_{xv} + \sum F_{xt}\cos\theta - \sum F_{yt}\sin\theta
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
m_v\left(\dot{V}_v + U_v\omega_v\right) + m_t\left(\dot{U}_t - V_t\omega_t\right)\sin\theta \\
+m_t\left(\dot{V}_t + U_t\omega_t\right)\cos\theta \\
= \sum F_{yv} + \sum F_{xt}\sin\theta + \sum F_{yt}\cos\theta
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
I_v\dot{\omega}_v + d_v\cdot m_v\left(\dot{V}_v + U_v\omega_v\right) \\
= \sum M_v + d_v\cdot\sum F_{yv}
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
I_t\dot{\omega}_t - d_t\cdot m_v\left[\left(\dot{U}_v - V_v\omega_v\right) - \left(\dot{V}_v + U_v\omega_v\right)\right] \\
= \sum M_t - d_t\cdot\left(\sum F_{xv}\sin\theta - \sum F_{yv}\cos\theta\right)
\end{aligned} \tag{4}
$$

where m_v, I_v, m_t and I_t are the masses and yaw moment of inertias of the vehicle and trailer, respectively. The motion of the vehicle/trailer is defined by longitudinal velocities U_v, V_v, lateral velocities U_t and V_t, and yaw rates ω_v and ω_t. The hitch angle $\theta = \psi_t - \psi_v$ is equivalent to the relative orientation of the trailer with respect to the vehicle. Lengths d_v and d_t denote the longitudinal distances from the Center of Mass (CoM) to the hitch points in the vehicle/trailer axes, respectively. $\sum F_{xv}$, $\sum F_{xt}$ are the longitudinal tire forces and $\sum F_{yv}$, $\sum F_{yt}$ are the lateral tire forces acting on the vehicle and trailer, respectively. $\sum M_v$ and $\sum M_t$ are the yaw moments about the CoM of the vehicle and trailer, respectively, due to tire forces. Note that vehicle steering angle δ_v is directly controlled by the driver, trailer steering angle δ_t is directly controlled by our controller, and the un-actuated hitch angle θ is indirectly controlled.

3. NON-HOLONOMIC MOTION OF ARTICULATED VEHICLES

The kinematics of slip-free steering of an articulated vehicle with a single-axle trailer is well known and studied [5], [11]. However, there is little existing literature when the trailer is dual-axle without a dolly and possesses a long wheelbase. Such a system, in which the trailer is *without* steering, can only move in a straight line under the no-slip condition, as the velocity of each tire contact patch must be aligned with the wheel's longitudinal axis (i.e. there is no lateral component of velocity for the contact patch). For such a system to move in a curved path, significant tire slip is necessary, requiring larger towing forces and potentially causing damage to the hitch, suspension and tires. In order to follow curved trajectories, not only do these types of trailers require steering, but the steering angles need to be continuously correlated with the steering angle of the vehicle and the hitch angle.

Movement under no-slip condition reduces wheel slip, effectively reducing wasted energy and minimizing tire wear. Note that dual-axle trailers with axles are situated close together longitudinally do not suffer from this problem and are able to behave almost similar to their single-axle counterparts with small amounts of slip. Long wheelbase dual-axle trailers with a front axle dolly introduces an additional yaw DoF, allowing the entire system to effectively behave as if there were two single-axle trailers inline. However the additional yaw DoF increases the difficulty of controlling the coupled system [12].

Extending the concept of no-slip kinematics of an articulated system, we can compute the necessary trailer steering angle (δ_t) and hitch angle (θ) for any given vehicle steering angle (δ_v) such that both vehicle and trailer move about the same instantaneous center of rotation (ICoR). This constraint of a common ICoR is necessary for ensuring the no-slip motion under the assumption that dynamic effects are minimal in low speed maneuvering. For this, first we define the *hitch control space*, which is the 3D Euclidean space spanned by the configuration variables $(\delta_v, \delta_t, \theta)$. Next we derive the equation of the *no-slip curve* for the articulated system, which is a curve in the hitch control space that contains all no-slip combinations of the configuration variables.

Assuming no-slip, for any vehicle steering angle δ_v, the radius r_v about the ICoR for the vehicle rear-axle midpoint is given by:

$$r_v = \frac{l_v}{\tan\delta_v} \tag{5}$$

where l_v is the vehicle wheelbase. Since the hitch point is common to both vehicle and trailer, it also must rotate about the same ICoR as the vehicle as well as the trailer. Based on the geometry of both vehicle and trailer, the corresponding radius r_t about the ICoR for the trailer front axle is given by:

$$r_t = \pm\sqrt{c_v^2 - c_t^2 + r_v^2} \tag{6}$$

where c_v and c_t are the hitch lengths for the vehicle and trailer, respectively. The positive square root of Equation (6) is taken for $r_v > 0$ and the negative square root for $r_v < 0$. The resulting necessary trailer steering angle δ_t for non-holonomy is given by:

$$\delta_t = -\arctan\left(\frac{l_t}{r_t}\right) \tag{7}$$

where l_t is the trailer wheelbase. The resulting equation of the no-slip curve, as determined through kinematic analysis, is given by:

$$\theta = \arctan\left(\frac{c_v\tan\delta_v}{l_v}\right) - \arctan\left(\frac{c_t\tan\delta_t}{l_t}\right) \tag{8}$$

For each δ_v there exists a unique (θ, δ_t) combination such that non-holonomy is preserved, and the kinematic no-slip condition is defined purely by the geometric properties of the vehicle and trailer. The no-slip curve for vehicle/trailer parameters given in Table 1 is shown in Fig. 3.

Table 1: 1:10 Scale model vehicle/trailer geometric parameters

Parameter	Description	Value
l_v	Vehicle wheelbase	270mm
c_v	Vehicle hitch length	82mm
T_v	Vehicle trackwidth	212mm
l_t	Trailer wheelbase	270mm
c_t	Trailer hitch length	146mm
T_{tf}	Trailer front trackwidth	212mm
T_{tr}	Trailer rear trackwidth	211mm

Our control objective is to operate the articulated system as close as possible to the no-slip curve for any given δ_v. At low speeds, when dynamic effects are not pronounced, we can use the no-slip curve as the objective to reduce slip (and thus energy losses) in the system for any motion.

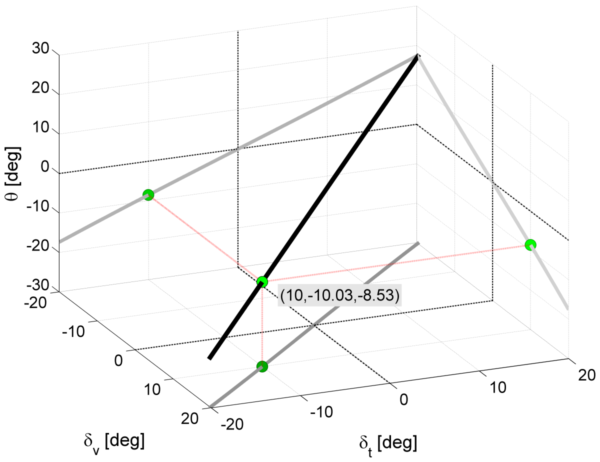

Fig. 3: The no-slip curve for an articulated vehicle equipped with rear wheel steering on the trailer, shown in the hitch control space, with point (δ_v = 10°, δ_t = -10.83°, θ = -8.53°). The curve is determined by the set of all no-slip combinations for the configuration variables $(\delta_v, \delta_t, \theta)$

4. STABILIZATION OF BACK-UP MOTION

The control goal is to drive the system from any initial point in the hitch control space to the no-slip curve. Once the system reaches the no-slip curve, the controller should continuously adjust the trailer steering angle to keep the system close to the no-slip curve regardless of changes in the vehicle steering input angle. The no-slip curve is used to generate the reference steering input for the trailer $\delta_{t,r}$ and the reference hitch angle θ_r for any given δ_v, and the articulated system is maneuvered into this configuration through the indirect manipulation of θ. However, merely bringing the system near the no-slip curve does not result in its stability because in backing-up motion the no-slip curve itself represents unstable behavior, as can be shown using eigenvalue analysis [1]. A single vehicle does not exhibit this unstable behavior in low speed reverse motion, however the addition of the trailer introduces unstable poles, identifiable by the root locus of the linearized equations (1)-(4) with $U_v < 0$. Increasing the reverse speed of the system increases the instability as seen by the unstable pole moving further right from the imaginary axis.

Through our proportional-integral controller with hitch angle feedback (Fig. 4) we attempt to make the no-slip curve a stable attractor and guide the articulated system towards it. In this control strategy, the vehicle steering angle δ_v is used to command a feed-forward trailer steering angle equal to the reference trailer steering angle $\delta_{t,r}$. The measured hitch angle θ is compared to the reference hitch angle θ_r to generate an error signal $\tilde{\theta} = \theta_r - \theta$ that is used in the PI feedback controller.

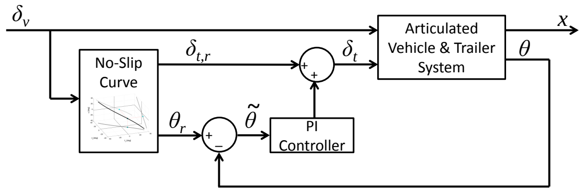

Fig. 4: Block diagram for closed loop driver assist controller using hitch angle feedback.

Eigenvalue analysis can again be used to verify the stabilizing qualities of our hitch angle feedback system, which shifts the single unstable pole to the left-half plane. The resulting controller does not depend on the system dynamics, but only on the geometric parameters of the wheelbase and the vehicle and trailer hitch lengths. Fortunately, for a laterally symmetric system, the no-slip curve is almost a straight line and it can be approximated as a linear relation for easier implementation on control hardware.

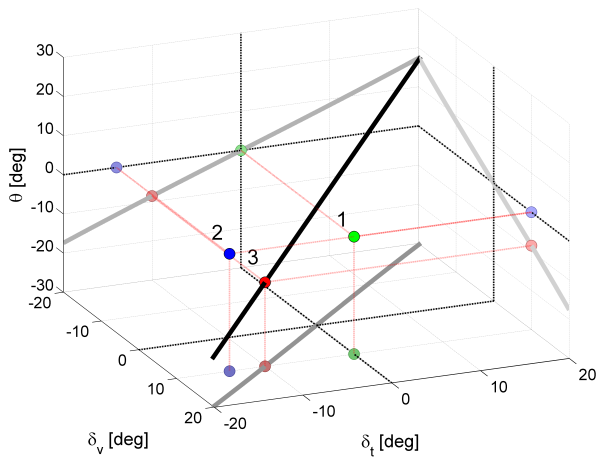

Fig. 5: Stabilization of the vehicle/trailer system from operating point 1 towards the no-slip curve (point 3) for fixed δ_v may require intermediate point 2 where the trailer steering angle δ_t exceeds the reference angle $\delta_{t,r}$

An example of the stabilization strategy during reverse motion is shown in Fig. 5, with the initial system configuration denoted by Point 1 and the desired final configuration of the vehicle-trailer is given by Point 3. From simulation and experiments we can verify that it is not always possible to go directly from Point 1 to Point 3. In this case the initial hitch angle is zero, however the desired hitch angle corresponding to $\delta_v = 10°$ is θ_r = -8.53°. Configurations near Point 1 on line 1 → 3 have a large δ_v but a small δ_t, resulting in a large (negative) vehicle yaw rate but a small trailer yaw rate, effectively increasing the hitch angle, which is the opposite of the desired direction. Furthermore, we cannot simply set δ_t to the reference value $\delta_{t,r}$ and begin driving in reverse because the goal is to obtain a larger yaw rate on the trailer than the vehicle such that θ decreases and goes towards θ_r = -8.53°.

We have found that the necessary stabilization strategy is to command a $|\delta_t| > |\delta_{t,r}|$ (the magnitude beyond the feed-forward steering angle is dependent on the hitch angle error and the hitch angle feedback gain, currently determined empirically) to drive θ towards the no-slip curve. In this example, we show that at Point 1 the driver commands a δ_v and for some $|\delta_t| > |\delta_{t,r}|$, θ decreases towards the no-slip curve, shown at Point 2. As the operating point moves towards the no-slip curve, δ_t is adjusted such that the final operating point lies on the curve (Point 3).

There may exist a situation where the necessary δ_t to move θ towards the no-slip curve exceeds the physical limit of δ_t, in which case a haptic, visual or aural feedback can be used to warn the driver of excessive δ_v for the given initial configuration. The limit of the necessary conditions for δ_t such that a stable (i.e. non-jackknife) motion can be calculated *a priori* via analysis of the dynamic system model, and is to be covered in a future publication.

By achieving reverse motion stabilization through trailer steering modulation only, it is possible to drive the articulated system without requiring prior trailer driving experience. Beyond providing stabilization of the reverse motion, traditional trailers require counter-intuitive steering inputs to initiate a turn, necessitating an initial steering input in the opposite direction to the intended direction. With our system it is possible to back-up the entire system using inputs similar to that of driving the vehicle without any trailer. An example of the difference in the necessary "idealized" steering input to perform a single turn is shown in Fig. 6. Note that to accomplish the same turn, the uncontrolled system requires an initial input of negative δ_v between states 1 and 2, and when the turn is completed the δ_v magnitude must be initially increased between states 4 and 5. The control system assists the driver by eliminating the necessity for such counter-intuitive inputs to the vehicle steering, made possible through the use of trailer steering to control the hitch angle and the trailer motion.

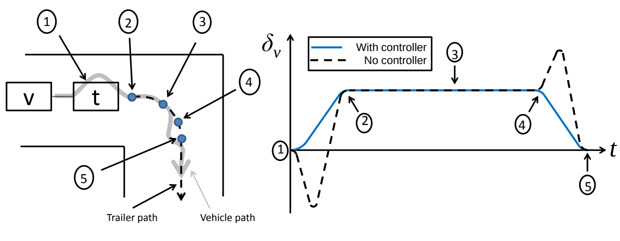

Fig. 6: Comparison of the steering profiles of a traditional trailer with the proposed control system for a 90° turn, highlighting vehicle deviation from the intended path for the uncontrolled system. Counter-intuitive input to enter and exit a turn is unnecessary with controlled system.

5. SIMULATION AND EXPERIMENTAL RESULTS

5.1 Simulation Model

The control algorithms are verified in simulation model using equations of motion (1)-(4) in Simulink. The vehicle is driven at a constant reverse speed (PI control of the vehicle longitudinal velocity U_v) and subjected to a variety of vehicle steering inputs. The closed loop system is able to modulate the trailer steering angle to drive the system to the no-slip curve and maintain it within close proximity as the vehicle steering angle continuously changes. We subject the system to a sinusodial steering input ($-10° \leq \delta_v \leq 10°$) for 1.25 cycles and then hold it at $\delta_v = 10°$, and observe that the trailer path behaves such that it follows the vehicle path, albeit leading the vehicle as the coupled system moves in reverse.

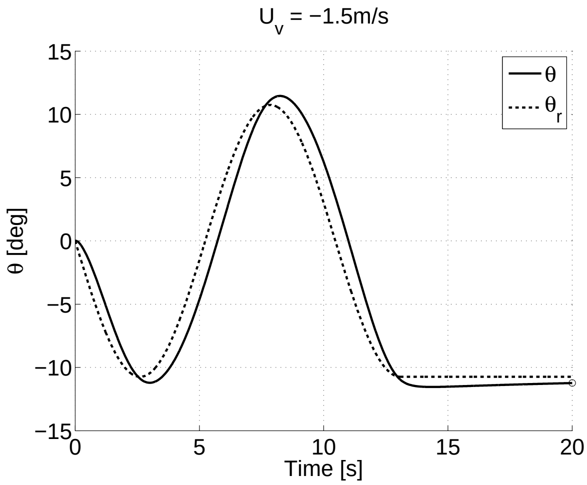

Fig. 7: Simulation of hitch angle tracking using the closed-loop control system with a varying vehicle steering input. The reference hitch angle is determined by steering angle and the no-slip curve.

Simulation data from Fig. 7 demonstrates the ability of the closed loop system to keep the hitch angle θ close to the desired reference hitch angle θ_r for the sinusoidal steering input. Because we cannot predict the driver's steering intent, the trailer steering has to respond to the driver steering inputs, thus θ appears to demonstrate a phase lag as the system reacts. The magnitude of θ is larger than θ_r as the control overshoots the reference hitch angle. However, if the steering angle is held for some duration, we can see θ tending towards θ_r.

5.2 Scale-Model Experimental Platform

The stabilization control is also experimentally verified on a 1:10 scale test platform. In this platform steering on both vehicle and trailer is achieved using servos, with commands sent via a wireless interface. The control algorithm is implemented in real-time on a PC using Matlab Real-Time Windows Target, controlled via a Logitech steering wheel/pedal input interface. The hitch angle is measured using an optical rotary encoder with the measured angle sent wirelessly to the host PC.

We demonstrate the effectiveness of the stabilization with complex maneuvers in both simulations and hardware tests, driving as if the operator were only driving the vehicle (Figs. 1 and 8). Without active stabilization it is extremely difficult to accomplish the slalom maneuver as there are several changes of directions, each requiring a counterintuitive steering input to initiate the turn, followed by a steering angle to modulate it about an equilibrium. The resulting motion is only completed after several cusps (change from reverse to forward motion to avoid jack-knifing before continuing), taking several minutes to reach the goal. However with our back-up control the maneuver is easily accomplished within 20 seconds or less, resulting in zero cusps in the motion, even for drivers of limited trailer driving experience. The behavior of the stabilized system is similar to that of a single vehicle on its own, with the trailer staying behind the vehicle.

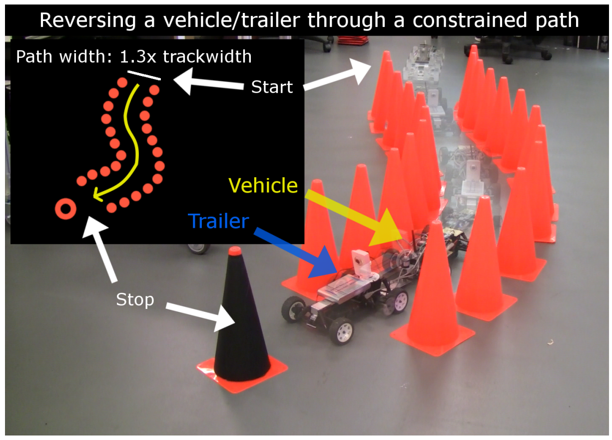

Fig. 8: Temporal sequence of experimental test: backing-up of an articulated system through a constrained path.

In the constrained path maneuver (Fig. 8) the construction cones are spaced such that the lateral width of the path is 1.3x the vehicle trackwidth, leaving little room for error. Without active stabilization and trailer steering it is not possible to complete this task since the necessary vehicle path deviation to initiate the trailer yaw angle is not realizable without contacting the cones. The motion of the operating point in the hitch control space is shown in Fig. 9. In this experiment the driver observes the motion of the vehicle/trailer and adjusts the steering of the vehicle. The trailer steering is automatically adjusted based on the control and stabilization strategy, allowing the driver to focus on driving the lead vehicle.

The initial operating point for the experiment is at $(\delta_v, \delta_t, \theta) = (2.02°, -1.29°, -0.25°)$, indicated by a ● in the hitch control space in Fig. 9. From there it follows a trajectory (traced by the thin line) around the no-slip curve. From the data, we show that the operating point stays near the no-slip curve for the duration of the maneuver. The final operating point near the no-slip curve is indicated by the ★ in the hitch control space, at $(\delta_v, \delta_t, \theta) = (-10.00°, 6.44°, 11.00°)$. The hitch angle error $\tilde{\theta} = \theta_r - \theta$ for this experiment is shown in Fig. 10, where a zero hitch angle error means that the measured hitch angle matches the desired hitch angle, the latter is determined by the no-slip condition. We observe only small hitch angle errors throughout the

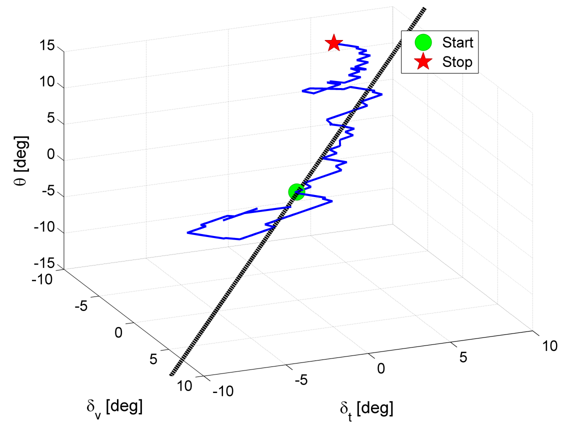

Fig. 9: The operating point of the closed-loop vehicle/trailer stays near the no-slip curve as the system is backing-up on a curve, for the experiment shown in Fig. 8.

entire maneuver, demonstrating the capability of the control system to guide the articulated system to the desired no-slip curve, all whilst the driver continually makes adjustments to the vehicle steering angle. To show the intuitive nature of the control system, we also show the vehicle steering angle as directly commanded by the driver via a steering wheel in Fig. 10. A positive steering input directly steers the articulated system counter-clockwise, whilst a negative steering input results in clockwise motion. Note that a vehicle without a trailer performing this maneuver would require a positive steering angle for the initial turn followed by a negative steering angle for the following turn. We are able to drive the coupled system using the same form of steering input, making the dynamic behavior more intuitive to drivers without experience in operating trailers.

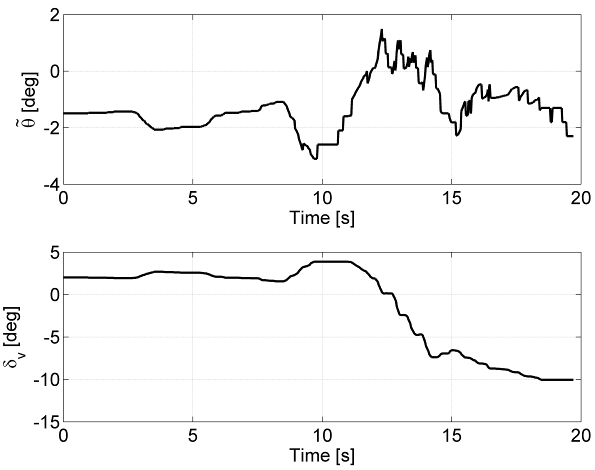

Fig. 10: Hitch angle error and vehicle steering input for the constrained path maneuver.

6. CONCLUSIONS

Using dynamic simulation and hardware experiments we have demonstrated the capability of the stabilized system to maneuver without requiring counterintuitive initial steering inputs to change the direction of the trailer, making the difficult task of backing up simple and intuitive to drivers without prior experience of driving such articulated systems. We introduced the concept of hitch control space, mapping the control inputs and target parameters for articulated vehicle motion, and provide the necessary reference steering and hitch angles for idealized no-slip motion via the no-slip curve. Using these concepts we show that stabilization can be achieved with simple PI feedback with trailer feedforward control. The proposed control strategy for trailer stabilization is achieved using only hitch angle and steering angle signals, allowing for easy adaption into vehicles of varying system parameters.

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